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On overall properties of micro-polar composites with interface effects

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1 Introduction

Size-dependence of mechanical property of composite materials has been well observed experimentally [1, 2]. Classical micro-mechanical models, however, fail to describe this phenomenon. There are two different ways to consider the size-dependency: (1) To include the nonlocal effect by idealizing the matrix material as a higher order continuum, e.g. micro-polar models [3] or strain gradient models. (2) To take into account the interface effect [4]. When the size of reinforced phase becomes small, both surface influence and nonlocal effect may become significant. In the present work, we combine these two approaches together by introducing the interface effect into micro-polar micro-mechanics.

2 Elastic effective properties of micro-polar composites with interface effects

The interface constitutive relations and the generalized Young-Laplace equation for micro-polar material models can be derived as

$$\mathbf{n}\cdot[\boldsymbol{\sigma}]\cdot\mathbf{P} = -\nabla_s\cdot\boldsymbol{\sigma}_s, \qquad \mathbf{n}\cdot[\boldsymbol{\sigma}]\cdot\mathbf{n} = -\boldsymbol{\sigma}_s:\mathbf{b}, \tag{1}$$

$$\mathbf{n}\cdot[\boldsymbol{\mu}]\cdot\mathbf{P} = -\nabla_s\cdot\boldsymbol{\mu}_s, \qquad \mathbf{n}\cdot[\boldsymbol{\mu}]\cdot\mathbf{n} = -(\boldsymbol{\mu}_s:\mathbf{b}+\boldsymbol{\sigma}_s:\mathbf{e}_s), \tag{2}$$

where $[\mathbf{f}] = \mathbf{f}^+ - \mathbf{f}^-$ stands for the jump of the variable $\mathbf{f}$. $\boldsymbol{\sigma}$ and $\boldsymbol{\mu}$ denote the unsymmetrical stress and moment stress tensors, respectively. $\mathbf{P}$ denotes the projections of strain and torsion tensors onto the tangent plane of the interface, $\mathbf{n}$ is unit normal of the interface, $\mathbf{b}$ is the curvature tensor of the interface, subscript s denotes the variables on the interface. The equation above are incorporated into the micro-polar micro-mechanical model to predict the overall elastic moduli of fiber-reinforced composites. In the present work, the special uniform tractions on boundary are adopted [3] and the symmetric average stress and average strain are redefined involving stress jump on interface. The symmetric part of the effective compliance tensor of the micro-polar composite, $\bar{\mathbf{M}}^{sym}$, can be obtained as

$$\bar{\mathbf{M}}_3^{sym} = \mathbf{M}_2^{sym} + f(\mathbf{M}_1^{sym} - \mathbf{M}_2^{sym}):\mathbf{P}_i^{sym} - f\mathbf{M}_2^{sym}:\mathbf{P}_s^{sym} \tag{3}$$

with $\mathbf{P}_i^{sym}$ as symmetrical stress concentration tensors of interface and $\mathbf{M}_\mathbf{i}^{sym}$ as local micro-polar compliances.

The variations of the effective shear modulus as functions of void radius are shown in Figs. 1a and 1b for the two types of surfaces: moduli of the surface of type I is positive, moduli of the surface of type II is negative. The predicted effective modulus increases with decreasing the void size characterized by $\boldsymbol{\delta}$. The material length $\boldsymbol{l_s}$ describes effects of interfaces. For the surface of type I (Fig.1a), nonlocal effects and surface effects are synchronized, and the effective shear modulus increases with the decreasing of the void size. It is found also that with the increase of $\boldsymbol{\delta}$, the size-effect is dominated by the nonlocal effect. For the surface of type II (Fig.1b), the size-dependency due to the nonlocal and surface effect are desynchronized, and the present theory (micro-polar with interface effect) predicts a decreasing effective shear modulus $\bar{\boldsymbol{\mu}}_\mathbf{3}$ when the void size is smaller than the critical value.

3 Plastic effective properties of micro-polar composite with interface effect

The perturbation law for micro-polar composite with interface effect was established by rigorous energy equivalence method, and it is used to estimate the average second order stress in local phases

$$\boldsymbol{\Sigma}^{sym}:\delta\bar{\mathbf{M}}_3^{sym}:\boldsymbol{\Sigma}^{sym} = <\boldsymbol{\sigma}:\delta\mathbf{M}:\boldsymbol{\sigma}+\boldsymbol{\mu}:\delta\mathbf{L}:\boldsymbol{\mu}>. \tag{4}$$

For a two-dimensional micro-polar material, the generalized Mises effective stress can be defined as [5]: $\boldsymbol{\sigma}_e^2 = \frac{3}{2}(\boldsymbol{\sigma}'_{(\alpha\beta)}\boldsymbol{\sigma}'_{(\alpha\beta)} + \frac{1}{l^2}\boldsymbol{\mu}_{\alpha 3}\boldsymbol{\mu}_{\alpha 3})$, which can be evaluated with help of equation (4). Plastic deformation will be developed in the matrix when the

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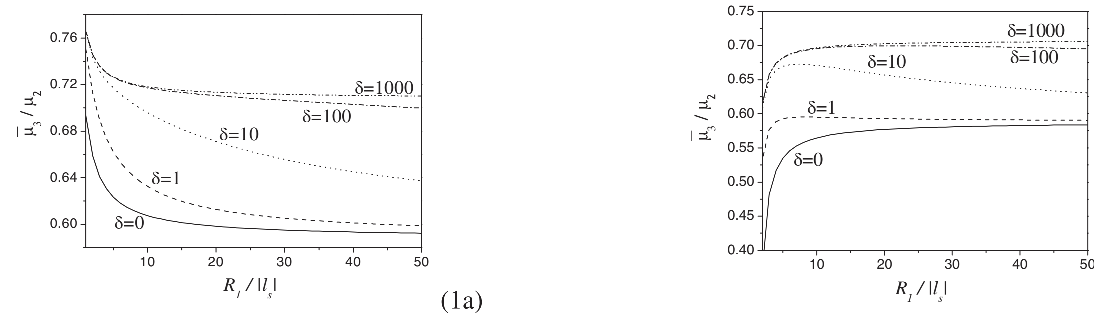
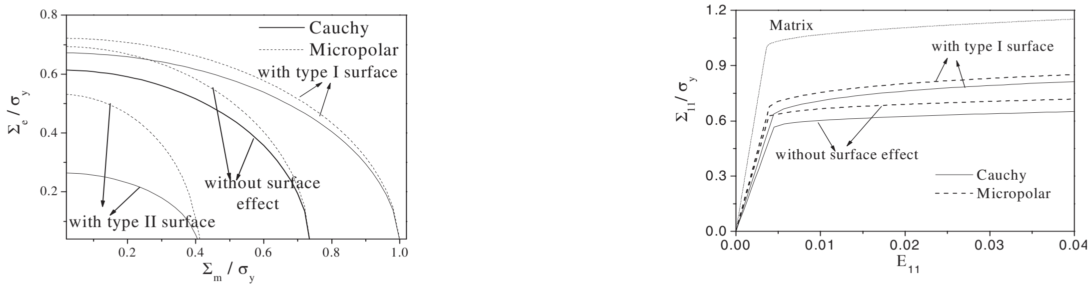


(1a) (1b)

Fig. 1 Effective shear modulus as a function of void radius characterized by δ for the surface of type I **(1a)** and type II **(1b)**.



(2a) (2b)

Fig. 2 Yielding surfaces predicted **(2a)** and uniaxial stress-strain curves predicted **(2b)** with or without micro-polar and surface effects.

applied macroscopic stress exceeds the initial yield stress. In order to consider the weakened constraint power of the plastic matrix on the fiber, the secant algorithm is proposed to predict overall elasto-plastic behavior for a micro-polar composite with interface effect [3]. For the micro-polar matrix, a power-law version of plasticity can be written with help of generalized effective stress defined as

$$\sigma_e = \sigma_y + h\epsilon_{ep}^n, \tag{5}$$

where h and n are plastic material constants. For a given plastic state σ_e, the secant moduli of the matrix material can then be defined as (in 2D situation):

$$\mu_0^s = \frac{1}{(1/\mu_0) + 3/\sigma_e[(\sigma_e - \sigma_y)/h]^{1/n}}, \ \ \kappa_0^s = \kappa, \ \ K_0^s = K_0 + \mu_0^s/3, \ \ (\beta_0^s + \gamma_0^s) = 2l^2\mu_0^s. \tag{6}$$

Some numerical calculations are performed in order to illustrate the previous theoretical formulations. It is found that the surface of type I or II strengthens or weakens the composite as shown in Fig.2(a), whereas micro-polar effect always predicts strengthening effects. Furthermore it is interesting to note that the interface effect leads to a significant size-dependence for a hydrostatic loading which can be related with the definition of the yield function, this is beyond discussions of the present work. The predicted macroscopic uniaxial stress-strain relations are given in Fig.2(b) for the surface of type I. It can be seen that for the surface of type I the interface effect leads to a significant strain hardening, especially for small void size δ. When the void size tends to be large, influence of the interface effect becomes small, then micro-polar effect dominates, finally classical prediction is recovered for infinite void size, as observed in elastic composites. The interface of type II can cause instability of the composite, and the unstable strain depends on the void size, its volume fraction and the weakening of the bulk materials.

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