Transparency Effect Induced by Elastic Metamaterials

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Abstract— In the quasi-static limit, it is shown that the neutral inclusion concept can be applied to predict the transparency phenomenon. The transparency designs for a coated ellipsoid for electromagnetic wave and a multilayered sphere for acoustic wave with metamaterials have been proposed. In this paper, we will show how to cloak a solid object with elastic wave metamaterial, where the shear wave in the system is nontrivial. Based on the Mie theory, it is shown that the effective bulk modulus, mass density, and shear modulus of the assemblage made of the coated spheres dominate the zero, the first and the second order scattering coefficients of one coated sphere, respectively. So letting the first three scattering coefficients vanish, the isotropic coated metamaterial can be determined in order to make a spherical object transparent for elastic wave, this again corresponds to the neutral inclusion concept.

1. INTRODUCTION

Since the pioneer work by Alù and Engheta [1], who found that plasmonic metamaterials could make a dielectric sphere with extremely low total scattering cross section, much works are devoted to analyze the transparency induced by metamaterials. In quasi-static limit, this phenomenon can be well predicted with the neutral inclusion concept [2], and it is easily applied for the more complex configurations, such as a coated ellipsoid and particulate composites. This method is suitable to cloak objects with dimension smaller than the operating wavelength. However, several transparent coated spheres joined together to form an object with large electrical size can still be transparent [3]. This may provide a new way to achieve transparency for an object with size larger than the wavelength. By analogy, the acoustic transparency for a coated sphere with acoustic metamaterials can also be designed [4]. Acoustic metamaterial is a kind of material, whose mass density and bulk modulus could be negative. The peculiar properties of this kind of material have been demonstrated experimentally [5, 6]. In this work, we will discuss how to cloak a solid spherical object using elastic wave metamaterials.

2. NEUTRAL INCLUSION CONCEPT

Consider a random-shaped region characterized by permittivity ε_* and permeability μ_* (or bulk modulus κ_*, shear modulus G_*, and mass density ρ_*) embedded in an infinite matrix with ε_0 and μ_0 (or κ_0, G_0 and ρ_0), the electromagnetic or acoustic waves propagate through this area as shown in Figure 1. The region can be made of either a homogeneous medium or a heterogeneous material. For the latter, ε_* and μ_* (or κ_*, G_* and ρ_*) then denote the effective material parameters of the heterogeneous material. It is not surprised that if the material property of this region is the same as that of the background medium (matrix), the electromagnetic or stress fields outside of this region will not be disturbed. In other word, the region will not be "seen" by an outside observer. When the region is made of a homogeneous material, this is a trivial case. However if the region is made

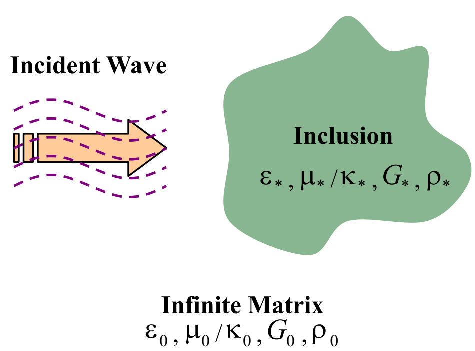

Figure 1: Scheme of neutral inclusion concept.

of a heterogeneous material, there are many design possibilities for equating its effective material property to that of the background medium, i.e., letting $\varepsilon_* = \varepsilon_0$ and $\mu_* = \mu_0$ (or $\kappa_* = \kappa_0$, $\rho_* = \rho_0$ and $G_* = G_0$). This is the basic idea of the "neutral inclusion" concept discussed extensively by Milton [7]. A neutral inclusion is a simple pattern (coated sphere, coated spheroid, etc.). When a neutral inclusion is embedded in a material made of assemblages of such pattern with gradual sizes (in order to fill the whole space), it will not perturb the static electric, magnetic, or mechanical fields outside of this inclusion. Although the neutral inclusion is defined in the static or quasistatic case, it can still provide useful information in the full-wave scattering case.

3. ELASTIC WAVE SPHERICAL CLOAK

Consider a coated sphere system characterized by bulk modulus κ_i, shear modulus μ_i, and mass density ρ_i, with the subscript $i = 1, 2, 3$ representing separately the sphere, the coating, and the host medium. Let r_1 denote the radius of the uncoated sphere and r_2 the radius of the coated sphere. A plane harmonic compressive wave propagates in the system. The total scattering cross section $Q_{\rm sca}$ of the coated sphere can be expressed as:

$$Q_{\rm sca} = \lambda_3^2 \sum_{n=0}^{\infty} \frac{1}{(2n+1)\pi} \left[|a_n|^2 + n(n+1)\frac{\alpha_3}{\beta_3}|b_n|^2 \right], \quad (1)$$

where α_3 and β_3 are propagation constants of longitudinal and transverse waves, respectively. $\lambda_3 = 2\pi/\alpha_3$ is the wavelength of the compressive wave in the host medium, a_n and b_n are the unknown scattering coefficients of scattered waves. Here we consider the host material has a nontrivial shear modulus, the coated sphere will scatter both P and S waves due to the coupling mode effect. In the Rayleigh limit, we have derived the first three scattering coefficients based on the Mie theory as follows:

Case I: *Solid* shell and *Solid* host material

$$\begin{aligned} a_0 &= i\frac{\kappa_{\rm eff}^{\rm HS} - \kappa_3}{3\kappa_{\rm eff}^{\rm HS} + 4\mu_3}(\alpha_3 r_2)^3,\ a_1 = \frac{\rho_{\rm eff}^{\rm M} - \rho_3}{3\rho_3}(\alpha_3 r_2)^3,\ a_2 = -\frac{20i\mu_3\left(\mu_{\rm eff}^{\rm HS} - \mu_3\right)/3}{6\mu_{\rm eff}^{\rm HS}(\kappa_3 + 2\mu_3) + \mu_3(9\kappa_3 + 8\mu_3)}(\alpha_3 r_2)^3, \\ b_1 &= -\frac{\rho_{\rm eff}^{\rm M} - \rho_3}{3\rho_3}\alpha_3\beta_3^2 r_2^3,\ b_2 = \frac{10i\mu_3\left(\mu_{\rm eff}^{\rm HS} - \mu_3\right)/3}{6\mu_{\rm eff}^{\rm HS}(\kappa_3 + 2\mu_3) + \mu_3(9\kappa_3 + 8\mu_3)}(\beta_3 r_2)^3. \end{aligned} \quad (2)$$

Case II: *Fluid* shell and *Solid* host material

$$\begin{aligned} a_0 &= i\frac{\kappa_{\rm eff}^{\rm HS} - \kappa_3}{3\kappa_{\rm eff}^{\rm HS} + 4\mu_3}(\alpha_3 r_2)^3,\ a_1 = \frac{\rho_{\rm eff}^{\rm B} - \rho_3}{3\rho_3}(\alpha_3 r_2)^3,\ a_2 = \frac{20i\mu_3}{3(9\kappa_3 + 8\mu_3)}(\alpha_3 r_2)^3, \\ b_1 &= -\frac{\rho_{\rm eff}^{\rm B} - \rho_3}{3\rho_3}\alpha_3\beta_3^2 r_2^3,\ b_2 = -\frac{10i\mu_3}{3(9\kappa_3 + 8\mu_3)}(\beta_3 r_2)^3. \end{aligned} \quad (3)$$

Case III: *Solid* shell and *Fluid* host material

$$a_0 = i\frac{\kappa_{\rm eff}^{\rm HS} - \kappa_3}{3\kappa_{\rm eff}^{\rm HS}}(\alpha_3 r_2)^3,\ a_1 = \frac{\rho_{\rm eff}^{\rm M} - \rho_3}{2\rho_{\rm eff}^{\rm M} + \rho_3}(\alpha_3 r_2)^3. \quad (4)$$

Case IV: *Fluid* shell and *Fluid* host material

$$a_0 = i\frac{\kappa_{\rm eff}^{\rm HS} - \kappa_3}{3\kappa_{\rm eff}^{\rm HS}}(\alpha_3 r_2)^3,\ a_1 = \frac{\rho_{\rm eff}^{\rm B} - \rho_3}{2\rho_{\rm eff}^{\rm B} + \rho_3}(\alpha_3 r_2)^3. \quad (5)$$

In every case, the parameter b_0 is not important and thus not given here.

The following parameters $\kappa_{\rm eff}^{\rm HS}$, $\mu_{\rm eff}^{\rm HS}$, $\rho_{\rm eff}^{\rm M}$ and $\rho_{\rm eff}^{\rm B}$ have been used, which are

$$\kappa_{\rm eff}^{\rm HS}/\kappa_2 = 1 + \frac{f(\kappa_1 - \kappa_2)}{\kappa_2 + (1-f)p(\kappa_1 - \kappa_2)},\ \mu_{\rm eff}^{\rm HS}/\mu_2 = 1 + \frac{f(\mu_1 - \mu_2)}{\mu_2 + (1-f)q(\mu_1 - \mu_2)}, \quad (6)$$

with $p = \frac{3\kappa_2}{3\kappa_2 + 4\mu_2}$, $q = \frac{6}{5}\frac{\kappa_2 + 2\mu_2}{3\kappa_2 + 4\mu_2}$,

$$\rho_{\rm eff}^{\rm M}/\rho_2 = 1 + f\frac{\rho_1 - \rho_2}{\rho_2},\ \rho_{\rm eff}^{\rm B}/\rho_2 = 1 + \frac{3f(\rho_1 - \rho_2)}{3\rho_2 + 2(1-f)(\rho_1 - \rho_2)}, \quad (7)$$

where $f = (r_1/r_2)^3$. For a composite filled with coated spheres that are randomly distributed in the host medium and have gradual sizes in order to fill the whole space, $\kappa_{\text{eff}}^{\text{HS}}$ and $\mu_{\text{eff}}^{\text{HS}}$ denote its effective bulk modulus and effective shear modulus calculated with the Hashin-Shtrikman (HS) bound [8]. $\rho_{\text{eff}}^{\text{M}}$ is the effective mass density obtained by the volume averaged method, whereas $\rho_{\text{eff}}^{\text{B}}$ is the effective mass density calculated with Berryman's formula [9]. From Eqs. (2) $\sim$ (5), we immediately get the transparency conditions $\kappa_{\text{eff}} = \kappa_3$, $\mu_{\text{eff}} = \mu_3$, and $\rho_{\text{eff}} = \rho_3$, which are consistent with those obtained directly from the neutral inclusion concept.

As an example, we employ an acoustic metamaterial to cloak an aluminum sphere (with a radius $r_1 = \lambda_3/5$) immersed in water. The cloaking metamaterial has the desirable material parameters $\kappa_2 = 0.47\kappa_3$ and $\rho_2 = 0.4\rho_3$, which slightly differ from the target value $\kappa_2 = 0.58\kappa_3$ and $\rho_2 = 0.55\rho_3$ predicted by the transparency conditions. There is a shifting effect when the quasi-static condition is used to predict the dynamic phenomenon. Figures 2(a) and 2(b) present the near field contour plots of the radial component of the scattered displacement field for an uncoated aluminum sphere and that with an optimized cloak, respectively. It can be seen that a sphere without the metamaterial cover leads to a strong and nonuniform scattering field in the fluid matrix, especially in the region adjacent to the sphere. However, when the cloaking metamaterial is employed as the cover, the scattering is dramatically reduced whilst the field strength within the cloak is large.

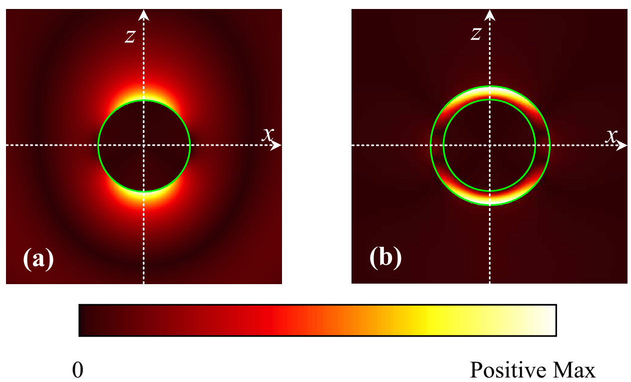

Figure 2: Contour plots of radial component of scattered displacement field for (a) uncoated aluminum sphere, and (b) same sphere with acoustic metamaterial.

4. CONCLUSIONS

With help of neutral inclusion concept, we derive the quasi-static transparency conditions for a solid system. By investigating scattering properties of the composite sphere, we find that the effective bulk modulus, mass density, and shear modulus of the assemblage made of the coated spheres dominate the zero, the first and the second order scattering coefficients of one coated sphere, respectively. These results can be used to obtain the transparency conditions, which agrees with those given by neutral inclusion concept. Numerical results in dynamic case have been given to confirm the proposed conditions.

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