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A SIMPLE TRANSVERSE DAMAGE MODEL FOR UNIDIRECTIONAL COMPOSITES

S. L. BAI†

Department of Mechanics and Engineering Science, Peking University, Beijing 100871, People's Republic of China

and

G. K. HU

Department of Applied Mechanics, Beijing University of Technology, Beijing 100081, People's Republic of China

Abstract—A simple transverse damage model for unidirectional composites was established, in the case when the composite specimen is loaded transversely. The model ignoring thermal stress, based on Griffith's virtual work argument and usual damage theories, gives a crack propagation criterion at the fiber/matrix interface. By extrapolation to the case of null crack length, the transverse tensile strength was found to depend directly on the crack opening angle at the interface and then on the interface surface energy. *In situ* transverse tensile tests were carried out on a unidirectional E-glass fibers/epoxy composite. The combination of the developed model with the experimental results gave the interface surface energy. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

FOR UNIDIRECTIONAL long fiber-reinforced composites, much effort has been made on the fracture [1, 2] and modeling, [3] when the composite specimens are loaded in the fiber direction. In these cases, based on the fiber fragmentation phenomena, the shear strength of the fiber/matrix interface could be estimated by using micromechanical models [4]. The shear-lag theories are often used. However, the transverse damage and fracture of unidirectional composites loaded perpendicularly to the fiber direction are not well known; especially those of the damage process and the de-bonding characterization of the fiber/matrix interface.

In fact, when a unidirectional composite specimen is loaded in the transverse direction, the transverse damage process could be divided, in general, into two steps: the first step is the de-bonding of the fiber/matrix interface at the specimen edges; the second is the crack propagation into the specimen until final rupture. The first step was studied theoretically in two dimensions by Toya [5], Ju [6] and Folias [7, 8]. By analyzing stress fields in vicinity of the crack tip, they proposed a criterion of crack propagation along the interface. However, the problem of crack propagation along the fiber length was not treated in their work. For a composite laminate in particular, the transverse cracking problem has been studied by other authors [9–11]. In this case, the cracking occurs in the 90° layers which are restricted by other layers. Therefore, their models can not be applied to a unidirectional composite.

In this paper, the transverse damage of unidirectional composites, loaded transversely, was studied theoretically and experimentally. A transverse damage model was proposed, with which the transverse modulus and tensile strength of unidirectional composites were evaluated and a crack propagation criterion was established by using Griffith's virtual work argument and damage concept. *In situ* transverse tensile tests were carried out in the interior of a scanning electron microscope. It should be noted that thermal stress introduced during the elaboration of composites was not taken into account in this work.

†To whom all correspondence should be addressed.

THEORETICAL ANALYSIS

Suppositions

A unidirectional composite specimen is loaded perpendicularly to the fiber direction as shown in Fig. 1. It is assumed that:

(1) The composite specimen is symmetrically damaged with two cracks propagating from two edges to the center.
(2) The cracks traverse through the specimen thickness.
(3) The cracks propagate at the fiber/matrix interface as shown in Fig. 1(c), which means that the interface is less resistant than the matrix.
(4) Thermal stress induced during the fabrication is neglected.

Under these conditions, the specimen could be considered to be composed of two parallel materials as shown in Fig. 1(b). Material I is the same as the original composite, material II is weakened by the cracks and is assumed to have zero modulus in the loading direction.

Transverse modulus of damaged composite

From knowing moduli of fiber and matrix and the fiber geometry, the moduli of the composites can be obtained with Mori–Tanaka's analysis: [12]

$$\mathbf{C} = \mathbf{C}_m\{I + V_f Q[I + V_f Q(\mathbf{S} - I)]^{-1}\}^{-1} \tag{1}$$

$$Q = [(\mathbf{C}_m - \mathbf{C}_f)(\mathbf{S} - I) - \mathbf{C}_f]^{-1}(\mathbf{C}_f - \mathbf{C}_m)$$

where, $\mathbf{C}_f$, $\mathbf{C}_m$, $\mathbf{C}$ are elastic tensors of the fiber, the matrix and the composite, respectively. $\mathbf{S}$ is Eshelby's tensor (see ref. [13]); V_f is the fiber volume fraction. $[X]^{-1}$ is the inverse matrix of $[X]$.

From Fig. 1(b), the transverse modulus of the damaged composite specimen can be written, by using the rule law, as $E_d = (1-\alpha)E_I + 2\alpha E_{II}$. E_d, E_I and E_{II} are Young's modulus of entirety, part I and part II of damaged composite specimen, respectively. Because of $E_I = E_t$, $E_{II} = 0$, we have

$$E_d = (1-\alpha)E_t \tag{2}$$

Here, $\alpha = \ell/W$ is called the damage parameter. E_t is the transverse modulus of composites without damage, and can be calculated with eq. (1). So, the transverse modulus of the damaged composite E_d depends linearly on the damage parameter α.

Criterion of crack propagation

According to Griffith's classical virtual work argument, if the end of a crack extends a small length $d\ell$, the change in the total energy of system is equal to the work needed to close up the crack to its original length. The crack is stable if this work is less than the energy required

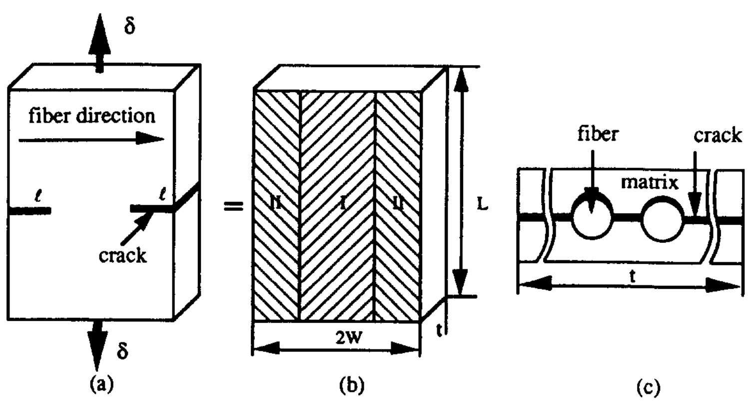

Fig. 1. Proposed transverse damage model of unidirectional composite specimen dimension: length L, width $2W$, thickness t.

to produce the new crack surface and is unstable otherwise. In equilibrium state, the critical strain energy release rate G is expressed as:

$$G = -(\mathrm{d}W_p/\mathrm{d}A + \mathrm{d}W_e/\mathrm{d}A) = 2\gamma \tag{3}$$

W_p is external work, W_e is elastic strain energy, γ is fracture surface energy. In the case of imposed displacement, $\mathrm{d}W_p = 0$, using $W_e = P\Delta/2$. Δ is imposed displacement, P corresponds to applied load and $\mathrm{d}A = 2t\mathrm{d}\ell = 2tw\mathrm{d}\alpha$. One obtains the following expression from eq. (3):

$$\Delta \mathrm{d}P = -8\gamma Wt\mathrm{d}\alpha \tag{4}$$

Now, by integrating eq. (4) and using $P = (1-\alpha)P_0$ [14] (P_0 is the load when $\alpha = 0$), and then replacing P by σ, we have the expression for the critical stress of crack propagation:

$$\sigma^2 = 2\gamma E_t(1-\alpha)^2/L \tag{5}$$

Here, Lemaitre and Chamboche's damage concept was used to obtain the relation between P and P_0. α is same as D defined by Lemaitre and Chamboche. Equation (5) gives a crack propagation criterion expressed by a linearly decreasing critical stress as a function of the damage parameter α.

Calculation of fracture surface energy

According to eq. (5), the critical stress depends on the fracture surface energy, γ, of the crack. If the crack propagates both at the fiber/matrix interface and in the matrix, this value could be expressed as $\gamma = V_i\gamma_i + V_m\gamma_m$ according to the rule law. γ_i and γ_m are the fracture surface energies of the interface and the matrix, respectively. V_i and V_m are the interface and matrix volume fraction, respectively.

Taking square array fiber distribution as an example, the expression of crack surface energy can be written as below (see Appendix A):

$$\gamma = \frac{1}{1-\varphi(\frac{d}{t}, V_f)(\theta - \sin\theta)}\left\{\theta\varphi(\frac{d}{t}, V_f)\gamma_i + \left[1 - \sin\theta\varphi(\frac{d}{t}, V_f)\right]\gamma_m\right\} \tag{6}$$

By introducing eq. (6) into eq. (5), we obtain the crack propagation criterion expressed as:

$$\sigma^2 = \frac{E_t}{L}(1-\alpha)^2\frac{1}{1-\varphi(\frac{d}{t}, V_f)(\theta - \sin\theta)}\left\{\theta\varphi(\frac{d}{t}, V_f)2\gamma_i + \left[1 - \sin\theta\varphi(\frac{d}{t}, V_f)\right]2\gamma_m\right\} \tag{7}$$

Equation (7) establishes the relation between the critical crack propagation stress σ and the interface surface energy γ_i. If the matrix surface energy γ_m and the crack opening angle α are known, one can obtain the interface surface energy γ_i by applying σ, obtained by calculation

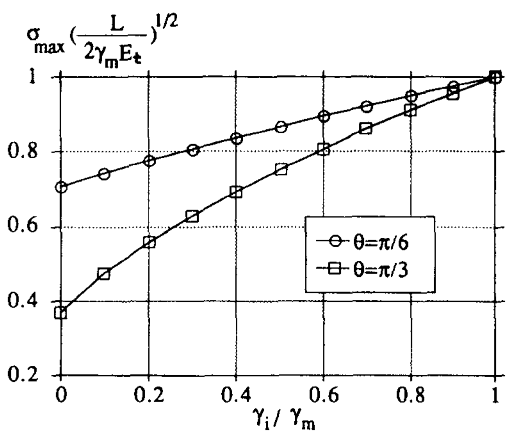

Fig. 2. Transverse tensile strength as a function of γ_i/γ_m.

with eq. (7). On the contrary, if the interface surface energy γ_i is known, one can estimate the critical crack propagation stress σ.

An important application of eq. (7) is when $\alpha = 0$, one can obtain the initial cracking stress σ_{max}, which is considered as the transverse tensile strength of unidirectional composites because it is the maximum value. Figure 2 shows the variation of transverse tensile strength as a function of γ_i/γ_m for two crack opening angles. It is seen that the tensile strength increases with the interface surface energy. The greater the interface surface energy, the greater the transverse tensile strength. Therefore, for a composite system, once the matrix is chosen, the transverse tensile strength depends only on the interface strength. The crack opening angle θ depends on the interface strength and decreases with increasing interface surface energy. The transverse tensile strength increases logically with decreasing crack opening angle θ.

TRANSVERSE TENSILE TESTS AND EXPERIMENTAL RESULTS

Composite studied and its curing

The studied composite material is a unidirectional long E-glass fiber/epoxy composite. The epoxy is a diglycidyl ether of bisphenol A (DGEBA) type resin from Dow Chemicals, with a 4, 4′-diamino-3,3′-dimethyldicyclohexyl methane (3DCM) curing agent from BASF Co. The mechanical properties of fiber and matrix are given below:

DGEBA–3DCM	E_m = 2.8 GPa	υ_m = 0.37	σ_r^m = 90 MPa
E-glass fiber	E_f = 74 GPa	υ_f = 0.22	σ_r^f = 2100 MPa

where, E_m, υ_m, σ_r^m and E_f, υ_f, σ_r^f are Young's modulus, Poisson's ratio and the tensile strength of the matrix and the fiber, respectively.

The composite was preimpregnated using a filament winding under a pression of 14 bar for 1 h. The obtained composite has a fiber volume fraction of 75%. This fiber content is higher than that usually used in industries; this high fiber content has no particular purpose. The fiber diameter measures about 20 μm.

Mechanical tests and results

The transverse tensile tests were carried out with a small motor-driven machine which is installed in the interior of a scanning electron microscope. Limited by the capacity of the machine, only the load was registered as a function of time. The specimen geometry is shown in Fig. 3. For observation, one edge of the composite specimens was polished and then metalized with gold before testing.

A representative tensile curve is shown in Fig. 4 from which the load rate can be measured. It is found that the studied composites have a linear behavior until the rupture, its transverse tensile strength is equal to 15.31 MPa. Before final rupture, no cracking was observed because of the rapid crack propagation.

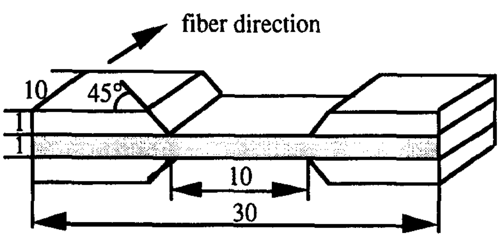

Fig. 3. Geometry of transverse tensile specimen (mm).

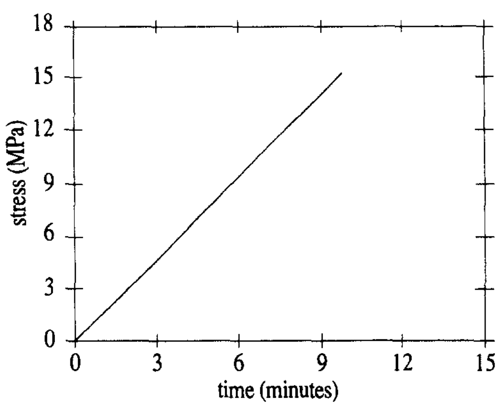

Fig. 4. Transverse tensile curve.

Fig. 5. Specimen rupture by transverse tensile tests.

Fig. 6. View of the interfacial debonding.

Figure 5 shows a picture of a specimen ruptured after testing. According to this picture, the cracking was along the fiber/matrix interface. The picture in Fig. 6 gives details of the interface and the matrix near the crack. The measurement of the crack opening angle was carried out on each fiber after enlargement. The measured average crack opening angle 2θ is about 91°.

DETERMINATION OF γ_i AND DISCUSSIONS

By introducing the measured tensile strength, average crack opening angle and other relative parameters into eq. (7), and letting $\alpha = 0$, the interface surface energy was found to be $2\gamma_i = 0.6\ \mathrm{kN\ m^{-1}}$ ($2\gamma_m$ is taken as $0.20\ \mathrm{kN\ m^{-1}}$ according to ref. [15]). Here, the interface debonding which took place somewhere near the macrocrack is not taken into account due to its small value.

The analytical results were obtained, ignoring the dissipated energy in the crack tip and the dynamic effects. The crack propagation criterion was established in a condition of imposed displacement. In fact, before the maximum stress is reached, the mechanical behavior of the composite is the same whatever the imposed condition may be (imposed force or imposed displacement [16]).

The fracture surface energy γ is a material constant in the case of static loading. This energy controls, in reality, the crack propagation. For composite materials reinforced with fibers, the cracking occurs often at the fiber/matrix interface owing to its imperfect properties. It is reasonable to treat γ as two parts γ_i and γ_m. The determination of γ_i is therefore very important for the design and fabrication of composite materials. Lacking any direct means for measuring γ_i, the model proposed can indirectly give an estimated value. The value γ_i for a given fiber/matrix system, should depend on local curing degree (or reaction degree between fiber and matrix). Therefore if a curing process is fixed, γ_i varies with V_f because local curing is affected by the fiber content. This can be explained as local curing process decides the interface surface energy γ_i. In our model, the cracking trace in matrix is assumed to be a straight line, that makes the calculation of γ easy. In fact, when V_f is great (as for our composites), V_i is much greater than V_m, so taking the matrix crack as a straightline cannot give much discrepancy.

This model is developed for the unidirectional composites. However, it is possible to extend this model to composite laminates in which the cracking takes place firstly in the 90° layers.

CONCLUSIONS

A transverse damage model was developed for unidirectional fiber-reinforced composites by combining the damage concept with the linear elastic fracture mechanics. Based on this model, a crack propagation criterion was established to give a critical crack propagation stress as a function of interface surface energy and crack opening angle. By extrapolation of this criterion to the case of $\alpha = 0$, one can estimate the transverse tensile strength of unidirectional composites.

The transverse tensile strength was found to be 15.31 MPa for studied composites according to the *in situ* transverse tensile tests. By combining the experimental results and model, the interface surface energy of studied composites was found to be $0.16\ \mathrm{kN\ m^{-1}}$.

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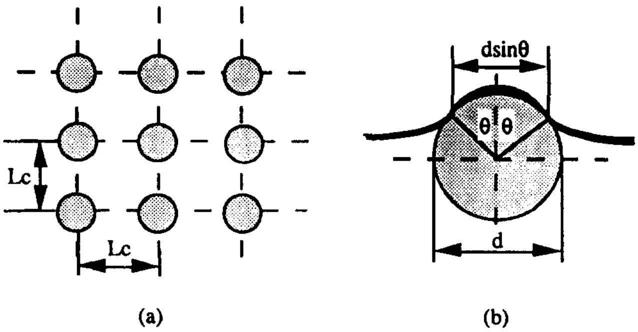

Fig. A1. (a) Square fiber distribution and (b) the opening angle 2θ of the interface.

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APPENDIX A

Calculation of fracture surface energy

Definitions:

n:	number of fibers transversed by one macro crack
2θ:	interface crack opening angle
d:	fiber diameter
L_c:	unit distance between two fibers.

One has the relations from Fig. A1:

$$V_f = \frac{\pi d^2}{4L_c^2}, \quad n = I\left(\frac{t}{L_c}\right) = I\left(\frac{2t}{d}\sqrt{\frac{V_f}{\pi}}\right)$$

$I(t/L_c)$ is the entire part of t/L_c (e.g. $I(1.6) = 1$). By using $V_i = L_i/(L_m + L_i)$ and $V_m = L_m/(L_m + L_i)$, here, $L_i = nd\theta$ is the debonding interface length and $L_m = nd\sin\theta$ is the matrix crack length. The total surface energy of crack has the following expression:

$$\gamma = \frac{1}{1 - \varphi(\frac{d}{t}, V_f)(\theta - \sin\theta)}\left\{\theta\varphi(\frac{d}{t}, V_f)\gamma_i + \left[1 - \sin\theta\varphi(\frac{d}{T}, V_f)\right]\gamma_m\right\}$$

where $\varphi(d/t, V_f) = \frac{d}{t} I(\frac{2t}{d} P_f)$.

Here, P_f is the fiber packing factor. For square fiber distribution, $P_f = \sqrt{V_f/\pi}$; for hexagonal fiber distribution, $P_f = \sqrt{\sqrt{3}V_f/2\pi}$, where ϕ $(d/t, V_f)$ is a function of d/t and V_f. It varies in a narrow band with d/t as shown in Fig. A2. But it increases gradually with V_f as shown in Fig. A3.

The crack surface energy depends on the crack opening angle θ. If $\theta = 0$, $\gamma = \gamma_{max} = \gamma_m$ there will be no debonding of the interface. If $\theta = \pi/2$, $\gamma = \gamma_{min}$ because of $\gamma_i < \gamma_m$.

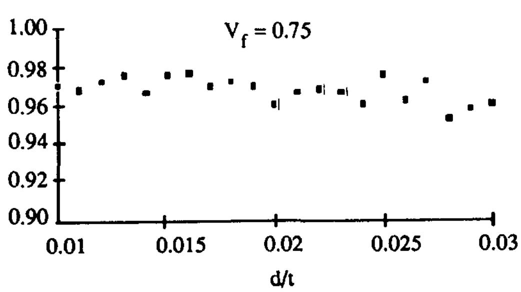

Fig. A2. Variation of $\varphi(d/t, V_f)$ as a function of d/t.

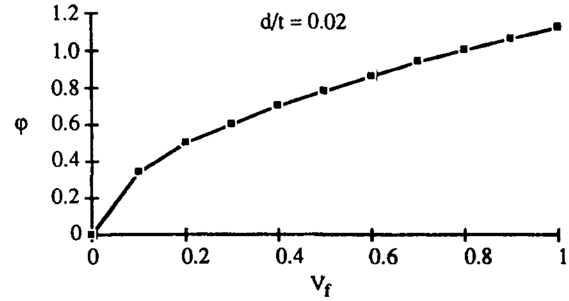

Fig. A3. Variation of $\varphi(d/t, V_f)$ as a function of V_f.

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