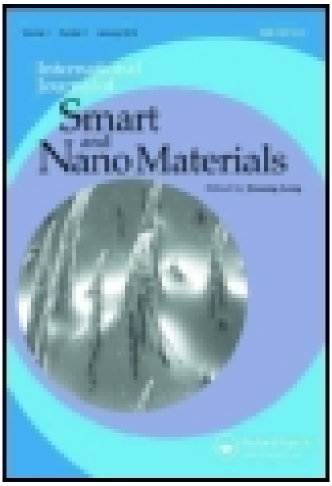

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Acoustic cloak constructed with thin-plate metamaterials

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We propose a strategy for designing the cylindrical acoustic cloak with thin-plate metamaterials. The inhomogeneous cloaking shell as derived by transformation acoustics is first discretized into a three-layer anisotropic metafluid, and their material parameters are optimized by minimizing the external scatterings. Then these metafluids are practically realized by thin-plate structures according to the metamaterial concept. As an example, an acoustic cloak is designed with nine layers of thin plate and totally 900 plate units. Numerical simulations are performed to assess the cloaking performance of the designed structure.

Keywords: acoustic cloak; thin-plate metamaterial; optimization

1. Introduction

Cloak is a device that keeps objects undetectable to electromagnetic waves or sound waves. This device is becoming true due to the rapid development of the transformation theory and metamaterial technology. Transformation theory is based on the invariance of coordinate transformation of wave system. Pendry et al. [1] first use this theory to design an invisibility cloak that can prevent electromagnetic radiation into the concealed region and cancel the outside scatterings in the meantime. The cloaking materials possess anisotropy and inhomogeneous material parameters. Based on the metamaterial technology, many structured prototypes of the electromagnetic cloak have been proposed and fabricated. Later, the cloaking concept has been extended to the acoustics realm and may open a novel application to acoustic stealth technology. Norris [2] demonstrates that, different from the invariance of Maxwell's equation, the transformation of acoustic wave equation is not unique. It means that acoustic cloaking material can be constructed with either the inertial metafluid with scalar compressibility and inertial mass tensor, or the pentamode material with scalar mass density and anisotropic compressibility. Focusing on the inertial acoustic cloak, Cheng et al. [3] design the cloak with concentric alternating fluid layers based on the effective medium theory. Daniel et al. [4] consider the discrete cloaking layer as being made of sonic crystals containing two types of solid cylinders immersed in fluid, whose elastic parameters should be properly chosen in order to satisfy the acoustic properties under request. Popa et al. [5] realize the ground acoustic cloak by acoustic metamaterials composed of blocks of steel, aluminum foam, and silicon carbide foam [6]. Zhang et al. [7] construct a quasi-two-dimensional inertial cloak with a network

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of acoustic circuit elements. Norris et al. [8] discuss the realization of the cloak region with only three acoustic fluids. More recently, the experimental realization of three-dimensional axisymmetric cloak [9] and ground cloak [10] has been reported.

A major issue of inertial acoustic cloak is the narrow frequency bandwidth induced by the dispersion effect of the resonant metafluid. A prospective solution of this issue is the active acoustic cloak with tunable operation frequency. This idea may be realized by active acoustic metamaterials, and their structural units are usually restricted to be the plate or shell equipped with piezoelectric devices. Regarding this issue, the purpose of this work is to develop a strategy for thin-plate acoustic cloak with the potential tunability mechanism. To do so, we first discretize the cloaking material with originally continuous parameter distribution into three effectively homogeneous metafluids. Material parameters of these effective metafluids are determined from the optimization process of minimum background scattering. This process is similar to what has operated for an electromagnetic cloak [11]. This part is presented in Section 2. We then consider thin-plate metamaterials with circular profiles, but still keep straight plates for each unit cells, in order to construct the cloaking layers. The procedure for designing thin-plate acoustic cloak and numerical verification of the cloaking effect are reported in Section 3.

2. Parameter optimization of acoustic cloak made of three-layer anisotropic metafluids

The parameter distribution of the cloaking material given by the transformation theory [12,13] is continuous and inhomogeneous in space. They are not readily realized with the structured metamaterials. To facilitate the structure design, the cloaking material with continuous parameter distribution is discretized into a three-layer anisotropic metafluid. We demonstrate below that the parameters of metafluids can be optimized to produce an imperfect cloak, but with the minor background scattering. The objective function of the optimization algorithm is the scattering cross section of the cloaked object. To this end, the analysis on a plane acoustic wave incident on a rigid cylinder coated with the cloaking material is conducted first.

2.1. Acoustic scattering by a circular cylinder coated with anisotropic metafluids

The cloaking model is shown in Figure 1, where a cylindrical scatterer to be concealed is cloaked by a three-layer metafluid with the scalar compressibility κ_i and anisotropic mass density $\tilde{\rho} = \mathrm{diag}\left[\rho_{r,i}, \rho_{\theta,i}\right]$ $(i = 1, 2, 3)$. The mass density and compressibility of the background medium (Region 4) are assumed as ρ_0 and κ_0.

The equations of motion in a medium with an anisotropic density can be written as [13]:

$$\nabla p = j\omega \begin{bmatrix} \rho_r & 0 \\ 0 & \rho_\theta \end{bmatrix} \rho_0 v \tag{1}$$

$$j\omega p = \kappa\kappa_0 \nabla \cdot v \tag{2}$$

Combine Equations (1) and (2) to get:

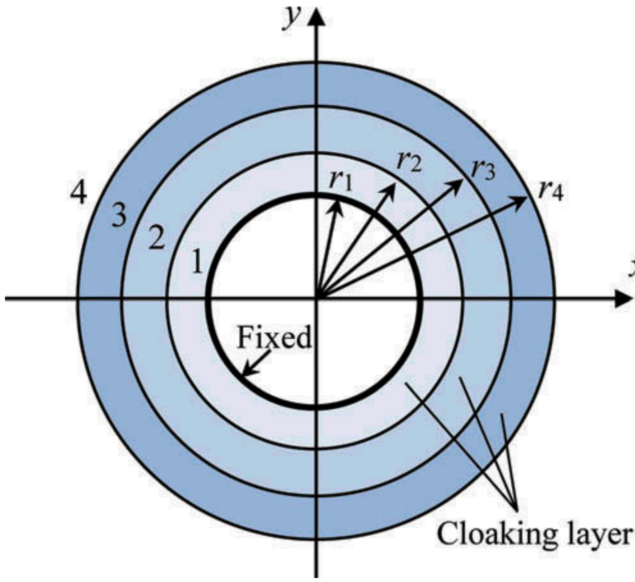

Figure 1. A rigid cylinder covered by three layers of homogeneous metafluid with anisotropic mass density.

$$\kappa\nabla\cdot\left(\begin{bmatrix}1/\rho_r & 0\\ 0 & 1/\rho_\theta\end{bmatrix}\nabla p\right)+k_0^2p=p, \tag{3}$$

where $k_0^2=\omega^2\rho_0/\kappa_0$. In the polar coordinate, Equation (3) can be rewritten as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{\rho_r}{r^2\rho_\theta}\frac{\partial^2 p}{\partial\theta^2}+\frac{\rho_r k_0^2 p}{\kappa}=0. \tag{4}$$

The method of separation of variables can be used to derive the general solution of Equation (4). Regarding the cloaking model under study, the pressure field $p^{(i)}$ in each region of the system can be written as:

$$\begin{cases} p^{(i)}(r,\theta)=\sum\limits_{m=0}^{\infty}\left[A_m^iJ_{q_i}\left(k_0r\sqrt{\frac{\rho_{r,i}}{\kappa_i}}\right)+B_m^iH_{q_i}\left(k_0r\sqrt{\frac{\rho_{r,i}}{\kappa_i}}\right)\right]e^{jm\theta}, r_i<r<r_{i+1}; i=1,2,3\\ p^{(4)}(r,\theta)=\sum\limits_{m=0}^{\infty}\left[A_m^4J_m(k_0r)+B_m^4H_m(k_0r)\right]e^{jm\theta}, r>r_4\end{cases}, \tag{5}$$

where J_{q_i} and H_{q_i} are Bessel and Hankel functions of the first kind with the order $q_i=m\sqrt{\rho_{r,i}/\rho_{\theta,i}}$. A_m^i (i = 1,2,3) and B_m^i (i = 1,2,3,4) are unknown scattering coefficients. A_m^4 is related to the incident plane wave and is given by:

$$A_m^4=\begin{cases}1, m=0\\ 2j^m, m>0\end{cases}. \tag{6}$$

The radial velocity can be written in terms of the pressure as:

$$v_r^{(i)} = \frac{1}{j\omega\rho_{r,i}\rho_{\theta,i}}\frac{\partial p^{(i)}}{\partial r}. \tag{7}$$

The continuity conditions of pressure and radial velocity of the system are:

$$p^{(i)}(r_{i+1}) = p^{(i+1)}(r_{i+1}), i = 1, 2, 3 \tag{8}$$

$$v^{(i)}(r_{i+1}) = v^{(i+1)}(r_{i+1}), i = 1, 2, 3. \tag{9}$$

At the boundary $r = r_1$, the fixed boundary condition is considered, namely

$$v^1(r_1) = 0. \tag{10}$$

All unknown coefficients A_m^i and B_m^i can be determined according to Equations (8)–(10). The scattering field in the background medium is given by:

$$P_{\mathrm{sc}} = \sum_{m=0}^{\infty} B_m^4 H_m(k_0 r)\cos(m\theta). \tag{11}$$

The scattering width (SW) of the coated cylinder is defined as:

$$\sigma = 2\pi R|P_{\mathrm{sc}}(\theta, R)/P_{\mathrm{inc}}|^2 \tag{12}$$

where P_{inc} is the incident pressure written as

$$P_{\mathrm{inc}} = J_0(k_0 r) + 2\sum_{m=1}^{\infty} j^m J_m(k_0 r)\cos(m\theta) \tag{13}$$

2.2. *Optimization of the cloaking parameters based on scattering cancellation*

The optimization process consists in searching the proper parameters of the cloaking layer for the least scatterings σ in the background. Considering the properties of Hankel function, the scattering field in the far field is inversely proportional to the distance, where scattering width σ becomes almost independent on the location under test. Consider also that the forward scattering ($\theta = 0$) is usually the largest among all directions. The goal of the optimization is defined as the minimum forward scattering in the far field region. The objective function σ is related to only the cloaking parameters, as follows:

$$\sigma(\theta = 0, R) = f(\rho_{ri}, \rho_{\theta i}, \kappa_i) \tag{14}$$

The optimization algorithm is based on the *fminsearch* function implemented in Matlab. Remind first that the continuous parameter distribution of the perfect cloaking layer [13] is given by:

$$\tilde{\rho}_{ri} = \frac{r}{r - r_1}, \tilde{\rho}_{\theta i} = \frac{r - r_1}{r}, \tilde{\kappa}_i = \left(\frac{r_4 - r_1}{r_4}\right)^2 \frac{r}{r - r_1}, i = 1, 2, 3. \tag{15}$$

The initial value, that is important for reducing the time cost in the optimization, can be taken as the value at the mid-point ($r = (r_i+r_{i+1})/2$) of the layer and expressed as:

$$\rho_{ri} = \frac{r_i + r_{i+1}}{r_i + r_{i+1} - 2r_1}, \rho_{\theta i} = \frac{r_i + r_{i+1} - 2r_1}{r_i + r_{i+1}}, \kappa_i = \left(\frac{r_4 - r_1}{r_4}\right)^2 \frac{r_i + r_{i+1}}{r_i + r_{i+1} - 2r_1}, i = 1, 2, 3. \tag{16}$$

As an example, three layers with r_2 = 0.4 m, r_3 = 0.5 m and r_4 = 0.6 m are used to cloak a rigid cylinder of radius r_1 = 0.3 m at 500 Hz. The density and bulk modulus of the air background are ρ_0 = 1.25 kg/m^3 and κ_0 = 0.147 MPa. The initial and optimized parameters of the cloaking layers relative to the background medium's parameters are listed in Table 1.

The total pressure fields for the uncloaked cylinder and cloaked one with initial and optimized parameters are shown in Figure 2. The numerical simulation is performed based on the commercial software package COMSOL Multiphysics PDE module. Compared with the scattering effect observed for the bare cylinder, the scattering has been efficiently weakened by the cloak having initial parameters (see Figure 2(b)). Furthermore, the optimized material parameters can lead to almost perfect cloaking effect, as shown in Figure 2(c). This example demonstrates that the optimization algorithm is efficient to improve the cloaking effect with only three coating layers.

Table 1. Initial and optimized material parameters in the cloaking region.

	Initial values			Optimized values		
	ρ_r	ρ_θ	κ	ρ_r	ρ_θ	κ
Layer 1	7	0.29	1.75	5.46	0.15	2.28
Layer 2	3	0.67	0.75	2.79	0.28	0.63
Layer 3	2.2	0.46	0.55	2.07	0.50	0.55

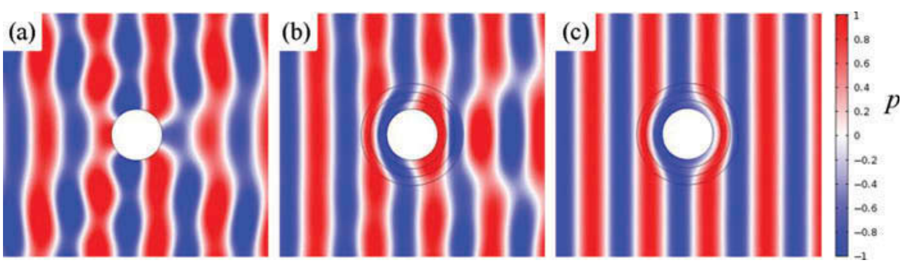

Figure 2. Total pressure fields at frequency 500Hz: (a) the cylinder without the cloak; (b) the cylinder with the cloak having initial parameters; (c) the cylinder with the cloak having optimized parameters.

3. Designing the cloaking structure with thin-plate metamaterials

3.1. Acoustic characteristics of thin-plate metamaterials with the circular profile

Thin-plate acoustic metamaterials are composed of parallel-stacked thin plates, attached periodically with local resonant units. A simple version of thin-plate metamaterials is the simply supported plate (see Figure 3(a)), and its effective acoustic property has been extensively analyzed regarding normally and obliquely incident waves [14]. It is found that the effective density of the simply supported plate is anisotropic, in which the radial density follows the Drude-medium model, while the annular density is almost the background one. It is also found that effective radial density won't vary with the change of the incident angle, when the wavelength of the incident wave is greater than the lattice constant of the plate. These properties make the thin-plate metamaterial a suitable candidate for the cloaking structure. To construct the cloaking layer, we change the straight shape of the plate into the circular profile, while keeping the plates straight in unit cells, as shown in Figure 3(b). The problem arising in the case of circular profile is that the length of straight plate locating in the different radial position will be different. But in order for contrasting an effective metafluid with a multilayered plate, it is convenient to make sure the same effective length of straight plates. The solution is that, the joint between plates at the innermost layer is fixed, and a part of the plate is fixed in other layers, so that the length of the free part of the plate is same to that of the innermost plate.

We assume and will verify later that the effective properties of the circular metamaterials inherit the ones of the straight plates. Consider the Young's modulus $E_Y = 0.12$ GPa, mass density $\rho_p = 1000$ kg/m^3, and Poisson's ratio $v = 0.33$ for the plate and the geometric parameters $L = 19.9$ mm, $d = 33$ mm, and the thickness of plate $h = 0.5$ mm. The background medium is the air with mass density $\rho_0 = 1.25$ kg/m^3 and sound velocity $c_0 = 343$ m/s. Figure 4 shows the density of a three-layer plate in the direction vertical to the plate surface, which is also the radial density in the circular model, as a function of frequency for different incident angles $\theta = 0, \pi/6, \pi/3$. It is seen that effective radial density is almost irrelevant to the incident angles below 500 Hz. We will show below that this angle-independent behavior is necessary for the cloak design.

To verify the effective properties of the circular model, we compute the scatterings of a rigid cylinder covered by the structured plates and their effective medium. For the plate structure considered in Figure 4, the thickness of the coating layer is set as 100 mm and the radius of the cylindrical core is 30 mm. At frequency 500 Hz, the effective parameters are $\rho_e = \mathrm{diag}[2.702, 1.015]\rho_0$ and $\kappa_e = 1.026\kappa_0$. The scattering width at $R = 2$ m for the two systems are shown in Figure 5(a). It can be seen that theoretical results based on

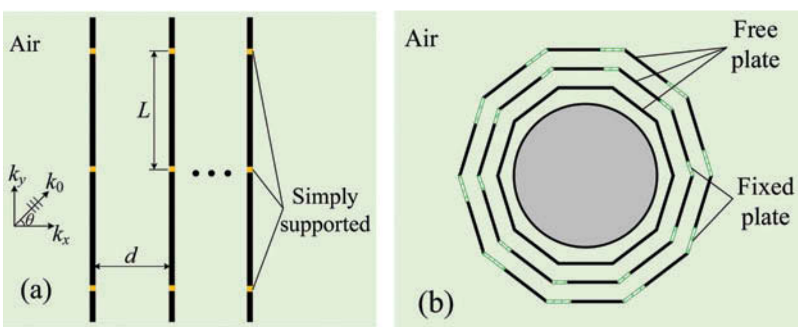

Figure 3. (a) The model of thin-plate acoustic metamaterials; (b) Schematic diagram of thin-plate metamaterials with the circular profile.

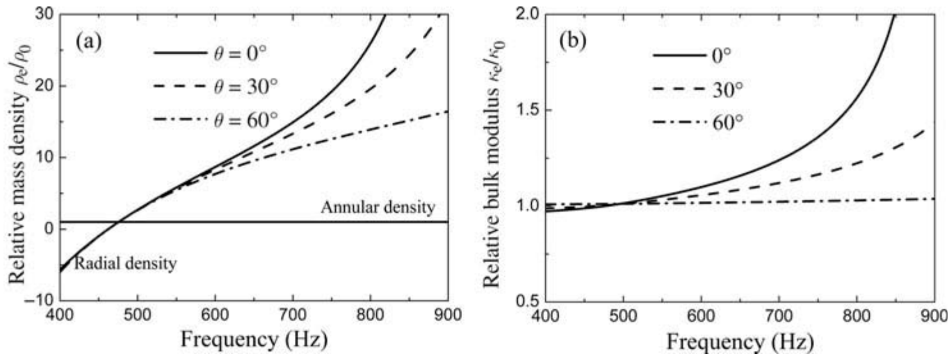

Figure 4. (a) Effective mass density and (b) effective bulk modulus of thin-plate metamaterials as a function of frequency at three different incident angles, θ = 0, π/6, π/3.

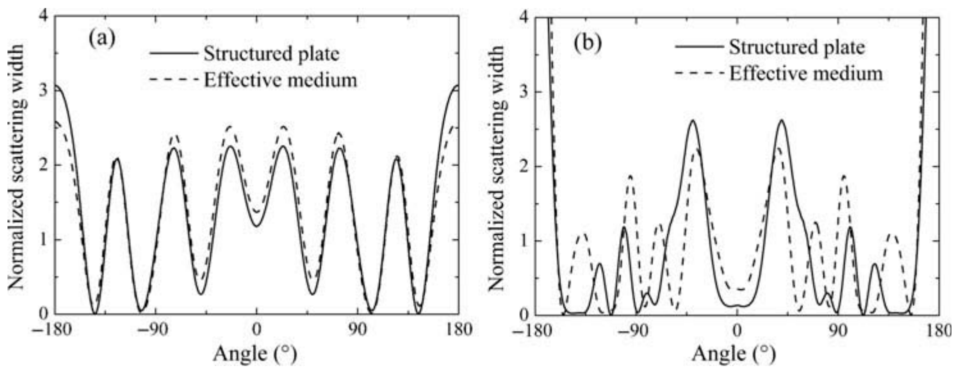

Figure 5. Normalized scattering width calculated at R = 2 m for the structured plate and their effective medium at frequencies (a) 500 Hz and (b) 800Hz.

effective parameters fit very well the scatterings of the actual structure. As a comparison, we show in Figure 5(b) the results of scattering width at frequency 800 Hz, where effective parameters are varied with respect to the incident angle. If effective radial density and modulus in normally incident case are considered, ρ_e = diag[26, 1.015]ρ_0 and $\kappa_e = 1.56\kappa_0$, the theoretical model gives a wrong prediction. Above results mean that the angle-independent behavior of the structured plate is necessary, considering the fact that the straight plates in the circular model must behave in the same manner when they interact with diffracted waves of any direction. In the next section, we will use the thin-plate structure to construct the acoustic cloak.

3.2. Structure design of acoustic cloak based on thin-plate metamaterials

Figure 6 shows the structure of acoustic cloak constructed with three groups of thin-plate structures; each group comprises three layers of plates to realize the effective fluid with the optimized parameters listed in Table 1. To add a new degree of freedom for tuning the annular density, the region between plates in each group is filled with a specific fluid different from the background medium. The number of plates for each layer is set as 100,

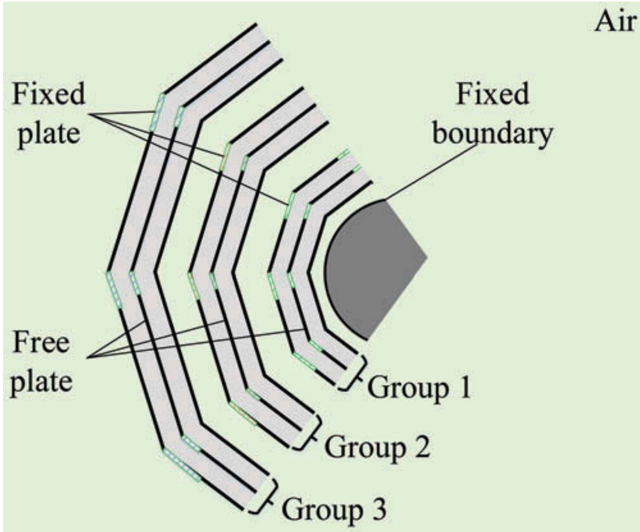

Figure 6. Schematic of the cloak structure made of thin-plate metamaterials.

then totally 900 thin plates are used to construct the acoustic cloak. For the targeting frequency 500 Hz, the corresponding air wavelength is about 20 times larger than the plate size. This ensures that the angle-dependent behavior of the plate will not appear in the current system.

The fluid filled between plates in each group plays the role of controlling the annular density and bulk modulus of the effective metafluid. Effective annular density of the N-layer plate system with the filling fluid is predicted by:

$$\frac{Nd}{\rho_{\text{eff}}} = \frac{(N-1)(d-h)}{\rho_{\text{filling fluid}}} + \frac{Nh}{\rho_{\text{plate}}} + \frac{d-h}{\rho_{\text{air}}}. \tag{17}$$

The desirable bulk modulus of the metafluid can be achieved by choosing the proper compressibility of the filling fluid. Once the filling fluid is finalized, the effective radial density is controlled by adjusting the thickness of the plate to fit the value required for the cloaking parameter. Note that this thickness variation has little influence on the annular density. For more detail, both effective modulus and radial density are predicted by transfer matrix method [14]. For an effectively homogeneous medium with anisotropic mass density $\tilde{\boldsymbol{\rho}}_{\mathbf{e}} = \text{diag}\left[\rho_x, \rho_y\right]$, the transfer matrix $\mathbf{T_e}$ of the pressure and normal velocity can be calculated analytically. The effective material parameters of thin-plate metamaterials can be retrieved by enforcing $\mathbf{T} = \mathbf{T_e}$, where $\mathbf{T}$ is the transfer matrix of the multi-layered plates and can be computed with numerical methods. The geometric and material parameters of the cloaking structure are listed in Table 2, which are designed to fulfill the optimized parameters shown in Table 1.

We perform numerical simulation to verify the cloaking effect of the designed structure. Figure 7(b) shows the pressure distribution of the cloak structure under a plane incident wave at 500 Hz. Compared with the uncloaked cylinder (see Figure 2(a)), the scatterings can be drastically reduced with the cloaking structure. The

Table 2. A set of geometric and material parameters of acoustic cloak constructed with thin-plate structures.

		Group 1	Group 2	Group 3
Plate	Length (mm)	19.9	26.2	32.5
	Thickness (mm)	0.5	0.5	0.5
	Distance between plates (mm)	33	33	33
	Mass density (10^3 kg/m^3)	1.5	0.5	0.5
	Young's modulus (GPa)	0.16	0.15	0.39
	Poisson's ratio	0.33	0.33	0.33
Filling fluid	Mass density (kg/m^3)	0.126	0.275	0.5
	Bulk modulus (MPa)	0.58	0.091	0.078

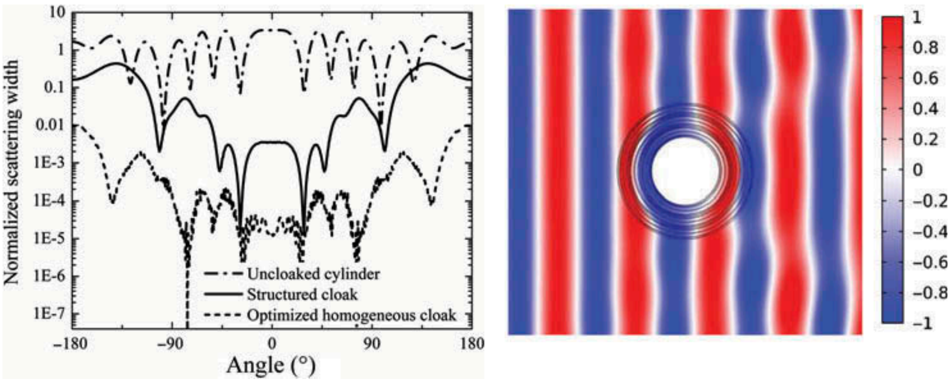

Figure 7. (a) Normalized scattering width of the uncloaked cylinder, the cylinder with the structured cloak, and that with optimized homogenous cloak; (b) Total pressure fields for the structured cloak incident by a plane acoustic wave.

scattering width calculated in the far field is presented in Figure 7(a) and shows that the scattering level has been reduced to 0.1 times less than the bare cylinder in almost all directions, even though the structured cloak is not as good as the imaginary cloak with optimized parameters.

We also calculate the total and back scattering widths at different frequencies, as shown in Figure 8(a) and (b), respectively. The total scattering can be suppressed in a narrow frequency band and reach a minimum value at the targeting frequency 500 Hz. This cloaking phenomenon is what we want to achieve by the parameter optimization and structure design. Due to the frequency-dependent properties of the plate structure, the narrow-band behavior of the cloak is inevitable, however, may be overcome by active thin-plate metamaterials with the tunable mechanism [15–17]. If the reduction of the wave reflection is only considered, the bandwidth of the back scattering width becomes wider in comparison with the total scattering results.

From the fabrication point of view, we can add constraints in the optimization procedure for pursuing physically realizable materials. However, it is inevitable that the annular density of the cloaking layer must be less than the background one because the concept of the parameter design is based on transformation acoustics. This brings the most difficult part of practical realization, as it is not easy to find a fluid with smaller density

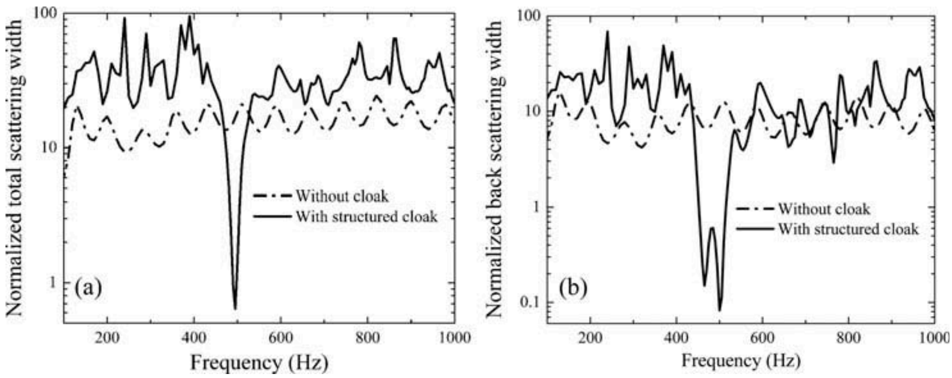

Figure 8. (a) Total scattering width and (b) back scattering width as a function of frequency in the absence and presence of the structured cloak.

than the air. However, it is worth to stress that the proposed approach for designing acoustic cloak is applicable to any fluid environment. If the background is water, it will be easier than in the air case to find a fluid material with density less than the water one. One example of this fluid can be the water filled with solid spherical shell containing the air.

4. Conclusions

In this work, we propose an idea to realize acoustic cloak with thin-plate metamaterials. The cloaking material is made of three layers of metafluids with anisotropic density. Using the optimization algorithm, the material parameters in each region of metafluids are determined with the goal of the minimized acoustic scatterings. The optimized parameters are anisotropic in density and can be realized by thin-plate metamaterials. As an example, a cloak is designed with nine layers of the plate structure and totally 900 plate units. Numerical simulation is conducted to verify the cloaking effect of the structure near the designed frequency. The proposed cloak based on thin-plate structures can be combined potentially with active piezoelectric devices [16], opening then a new direction for active control of acoustic cloaking in a wide frequency band.

Disclosure statement

No potential conflict of interest was reported by the authors.

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