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Dynamic effective models of two-dimensional acoustic metamaterials with cylindrical inclusions

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Abstract Based on analytical solutions of elastic waves scattered by a coated cylinder in an infinite elastic matrix, we construct the localization relations for averaged displacement and stress fields in each phase. Dynamic effective mass, in-plane bulk modulus and shear modulus are defined, respectively, as the ratio between the force and acceleration, bulk stress and bulk strain, maximum shear stress and maximum shear strain. Analytic expressions for dynamic effective parameters of two-dimensional acoustic metamaterials are derived. Numerical examples are given to analyze dynamic effective properties of composites with coated inclusions. It is demonstrated that the proposed model can predict negative values of effective mass and effective bulk and shear modulus, and discover the underlying mechanisms of negative effective material parameters. The proposed model will be helpful in designing new acoustic metamaterials.

1 Introduction

Acoustic metamaterials with local resonances have negative effective material parameters in the long-wavelength regime. Acoustic metamaterials with unusual dynamic properties can be used to block low-frequency noises [1–4], produce sub-wavelength images beyond the diffraction limit [5,6] and cloak objects without scattering [7,8], etc. To explore the exotic engineering applications, it is very important to predict effective dynamic properties of metamaterials and understand the underlying mechanisms how anomalous dynamic properties are realized. Effective dynamic properties can be determined by the transmission and reflection method [9], which retrieves effective parameters from a homogeneous material with the transmission and reflection spectra of metamaterials. The method can accurately predict effective dynamic parameters, but fail to disclose the physical mechanisms of the local resonance. It has been discovered from a discrete mass-spring model [10–12] that negative effective mass comes from the out-of-phase motion of the internal mass with respect to its surrounding material. For locally resonant sonic crystals with rubber-coated lead spheres in an epoxy matrix [1], the homogenization method has been presented to explain the "negative mass" [13]. But the method cannot give a prediction for higher-order resonant effects. So, it is necessary to develop a dynamic effective model to both predict effective properties and explore the "negative" nature of these effective material parameters in order to provide a design guide for acoustic metamaterials.

The anomalous overall properties of metamaterials result from the locally resonant effects of their building unit. This allows the prediction of the effective property by the dynamic effective model based on the single inclusion theory [14]. In a previous work [15], localization relations are constructed for averaged fields in each phase, based on analytical solutions of an elastic wave scattered by a coated sphere in an infinite elastic matrix. The effective mass, bulk modulus and shear modulus are defined, respectively, as the ratio between the force

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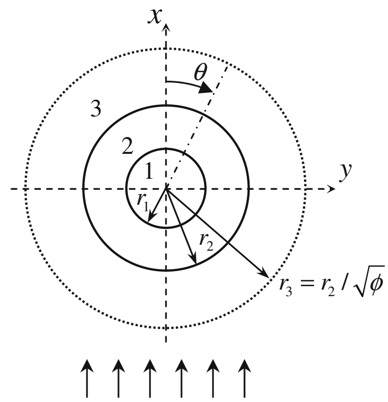

Fig. 1 The analyzed model

and acceleration, bulk stress and bulk strain, maximum shear stress and maximum shear strain. It is found that negative effective mass is induced by a negative acceleration field of the composite under a positive force. The negative effective bulk modulus appears for composites with an increasing (decreasing) total volume under a compressive (tensile) stress. The negative effective shear modulus describes composites with axisymmetric deformation under an opposite axisymmetric loading.

In this work, we will derive the analytic expressions of effective mass, in-plane effective bulk and shear moduli for two-dimensional (2D) acoustic metamaterials with cylindrical inclusions. First, analytic solutions for elastic waves scattered by a coated cylinder are given. Then, we propose the analytic expression for effective mass, in-plane effective bulk and shear modulus based on the averaged displacement and stress fields. Numerical results demonstrate that the proposed model can predict 2D composites with negative effective mass, bulk and shear modulus. These findings will help us design efficient low-frequency noise barriers and vibration dampers.

2 Scattering solutions of a coated cylinder

The analyzed model is a three-phase composite consisting of coated cylinders embedded in a host material. The building unit is a doubly coated cylinder, having the radii r_1, r_2 and r_3, respectively, for the uncoated cylinder, the coated cylinder and the outer boundary, as shown in Fig. 1. The matrix material covers the region with radius ranging from r_2 to $r_3 = r_2/\sqrt{\phi}$, where ϕ is the filling fraction of the coated cylinders. Each region is assumed to be elastic material characterized by mass density ρ_i, Lamé coefficients λ_i and μ_i with the subscript $i = 1, 2, 3$ representing separately the core, the coating and the host.

The scattering fields in the ith region are expressed as [16]

$$u_r^{(i)} = \sum_n u_{r,n}^{(i)}(r) \cos n\theta, \tag{1a}$$

$$u_\theta^{(i)} = \sum_n u_{\theta,n}^{(i)}(r) \sin n\theta \tag{1b}$$

for the displacements, where

$$u_{r,n}^{(i)}(r) = \frac{1}{r}\left(E_{11}^{(i)} a_n^{(i)} + E_{12}^{(i)} b_n^{(i)} + E_{13}^{(i)} c_n^{(i)} + E_{14}^{(i)} d_n^{(i)}\right), \tag{2a}$$

$$u_{\theta,n}^{(i)}(r) = \frac{1}{r}\left(E_{21}^{(i)} a_n^{(i)} + E_{22}^{(i)} b_n^{(i)} + E_{23}^{(i)} c_n^{(i)} + E_{24}^{(i)} d_n^{(i)}\right), \tag{2b}$$

with

$$E_{11}^{(i)} = nH_n(\alpha_i r) - \alpha_i r H_{n+1}(\alpha_i r),$$
$$E_{12}^{(i)} = nH_n(\beta_i r),$$

$$\begin{aligned}
E_{13}^{(i)} &= nJ_n(\alpha_i r) - \alpha_i r J_{n+1}(\alpha_i r),\\
E_{14}^{(i)} &= nJ_n(\beta_i r),\\
E_{21}^{(i)} &= -nH_n(\alpha_i r),\\
E_{22}^{(i)} &= -nH_n(\beta_i r) + \beta_i r H_{n+1}(\beta_i r),\\
E_{23}^{(i)} &= -nJ_n(\alpha_i r),\\
E_{24}^{(i)} &= -nJ_n(\beta_i r) + \beta_i r J_{n+1}(\beta_i r),
\end{aligned}$$

and

$$\sigma_{rr}^{(i)} = \sum_n \sigma_{rr,n}^{(i)}(r)\cos n\theta, \tag{3a}$$

$$\sigma_{r\theta}^{(i)} = \sum_n \sigma_{r\theta,n}^{(i)}(r)\sin n\theta \tag{3b}$$

for the stresses, where

$$\sigma_{rr,n}^{(i)}(r) = \frac{2\mu_i}{r^2}\left(E_{31}^{(i)}a_n^{(i)} + E_{32}^{(i)}b_n^{(i)} + E_{33}^{(i)}c_n^{(i)} + E_{34}^{(i)}d_n^{(i)}\right), \tag{4a}$$

$$\sigma_{r\theta,n}^{(i)}(r) = \frac{2\mu_i}{r^2}\left(E_{41}^{(i)}a_n^{(i)} + E_{42}^{(i)}b_n^{(i)} + E_{43}^{(i)}c_n^{(i)} + E_{44}^{(i)}d_n^{(i)}\right), \tag{4b}$$

with

$$\begin{aligned}
E_{31}^{(i)} &= (n^2 - n - \beta_i^2 r^2/2)H_n(\alpha_i r) + \alpha_i r H_{n+1}(\alpha_i r),\\
E_{32}^{(i)} &= n[(n-1)H_n(\beta_i r) - \beta_i r H_{n+1}(\beta_i r)],\\
E_{33}^{(i)} &= (n^2 - n - \beta_i^2 r^2/2)J_n(\alpha_i r) + \alpha_i r J_{n+1}(\alpha_i r),\\
E_{34}^{(i)} &= n[(n-1)J_n(\beta_i r) - \beta_i r J_{n+1}(\beta_i r)],\\
E_{41}^{(i)} &= n[\alpha_i r H_{n+1}(\alpha_i r) - (n-1)H_n(\alpha_i r)],\\
E_{42}^{(i)} &= -(n^2 - n - \beta_i^2 r^2/2)H_n(\beta_i r) - \beta_i r H_{n+1}(\beta_i r),\\
E_{43}^{(i)} &= n[\alpha_i r J_{n+1}(\alpha_i r) - (n-1)J_n(\alpha_i r)],\\
E_{44}^{(i)} &= -(n^2 - n - \beta_i^2 r^2/2)J_n(\beta_i r) - \beta_i r J_{n+1}(\beta_i r).
\end{aligned}$$

α_i and β_i are, respectively, longitudinal and transverse wave vectors. When a plane longitudinal (P) wave is incident on a coated cylinder, $c_0^{(3)} = 1$, $c_n^{(3)} = 2i^n$ $(n \geq 1)$, and $d_n^{(3)} = 0$. In the inner region $r \leq r_1$, $a_n^{(1)} = b_n^{(1)} = 0$. At the interfaces $r = r_1$ and $r = r_2$, the normal and tangential components of the displacement and stress fields should be continuous. Eight equations can be constructed to determine uniquely eight unknown scattering coefficients $c_n^{(1)}$, $d_n^{(1)}$, $a_n^{(2)}$, $b_n^{(2)}$, $c_n^{(2)}$, $d_n^{(2)}$, $a_n^{(3)}$ and $b_n^{(3)}$.

3 Effective dynamic mass

For time harmonic ($e^{-i\omega t}$) waves, the equilibrium equation of elastic materials is written as

$$\nabla \cdot \boldsymbol{\sigma} = -\rho\omega^2 \mathbf{u}. \tag{5}$$

Integrate Eq. (5) on a circular region S of the radius R, one obtains

$$\mathbf{F} = -\rho\omega^2 \bar{\mathbf{u}}, \tag{6}$$

where the Green formula has been used, $\mathbf{F} = \int \mathrm{d}l \cdot \sigma$ is the net force loading on the circular region, and the overall displacement is defined to be $\bar{\mathbf{u}} = \int \mathbf{u}\mathrm{d}S$. By use of Eqs. (3a,3b), it is found that the net force does not vanish only in the incident direction and is given by

$$F = R \sum_n \left[\sigma_{rr,n}(R)\, l_n - \sigma_{r\theta,n}(R)\, m_n\right], \tag{7}$$

where

$$l_n = \int_0^{2\pi} \cos n\theta \cos\theta \mathrm{d}\theta, \tag{8a}$$

$$m_n = \int_0^{2\pi} \sin n\theta \sin\theta \mathrm{d}\theta. \tag{8b}$$

Based on the orthogonal property of trigonometric functions, the nonzero solutions of Eqs. (8a,8b) are $l_1 = m_1 = \pi$. Therefore, the force F acting on the circular region of radius R can be computed by

$$F(R) = \pi R \left[\sigma_{rr,1}(R) - \sigma_{r\theta,1}(R)\right]. \tag{9}$$

According to Eq. (6), the macroscopic equilibrium equation for each region of a doubly coated cylinder can be written as

$$F(r_1) = -\rho_1 \omega^2 \bar{u}^{(1)}, \tag{10a}$$

$$F(r_2) - F(r_1) = -\rho_2 \omega^2 \bar{u}^{(2)}, \tag{10b}$$

$$F(r_3) - F(r_2) = -\rho_3 \omega^2 \bar{u}^{(3)}, \tag{10c}$$

where $\bar{u}^{(i)}$ denotes the averaged displacement of the ith region. Effective mass density ρ_{eff} can be defined as

$$\rho_{\mathrm{eff}} = -F(r_3) \bigg/ \left(\omega^2 \sum_i \bar{u}^{(i)}\right). \tag{11}$$

Substitute Eqs. (10a,10b,10c) into Eq. (11) to obtain the effective mass density of the three-phase composite

$$\rho_{\mathrm{eff}} = \frac{F(r_3)}{F(r_1)(1/\rho_1 - 1/\rho_2) + F(r_2)(1/\rho_2 - 1/\rho_3) + F(r_3)/\rho_3}. \tag{12}$$

4 Effective elastic properties

The constitutive equation of an isotropic linear elastic material is written as

$$\boldsymbol{\sigma} = 2\lambda\varepsilon_{\mathrm{b}}\mathrm{I} + 2\mu\boldsymbol{\varepsilon}, \tag{13}$$

where the bulk strain $\varepsilon_{\mathrm{b}} = (1/2)\,\mathrm{tr}\boldsymbol{\varepsilon}$ and the strain tensor $\boldsymbol{\varepsilon}$ are related to the displacement field $\mathbf{u}$ by

$$\varepsilon_{\mathrm{b}} = \frac{1}{2}\nabla \cdot \mathbf{u}, \tag{14a}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla). \tag{14b}$$

Integrate Eq. (14a) on a circular region S of the radius R, and use Eqs. (2a,2b) to obtain

$$\bar{\varepsilon}_{\mathrm{b}}(R) = \frac{R}{2} \sum_n \left[u_{r,n}(R)\, s_n\right], \tag{15}$$

where $\bar{\varepsilon}_\mathrm{b} = \int \varepsilon_\mathrm{b} \mathrm{d}S$ and

$$s_n = \int_0^{2\pi} \cos n\theta \mathrm{d}\theta. \tag{16}$$

In Eq. (16), the nonzero solution is $s_0 = 2\pi$. Therefore, the averaged bulk stress $\bar{\sigma}_\mathrm{b} = (1/2)\,\mathrm{tr}\boldsymbol{\sigma}$ in the ith region can be calculated by

$$\bar{\sigma}_\mathrm{b}^{(i)} = 2\kappa_i \bar{\varepsilon}_\mathrm{b}^{(i)}, \tag{17}$$

where $\kappa_i = \lambda_i + \mu_i$ is the bulk modulus. The effective bulk modulus κ_eff can be defined as

$$\kappa_\mathrm{eff} = \sum_i \bar{\sigma}_\mathrm{b}^{(i)} \bigg/ \left(2 \sum_i \bar{\varepsilon}_\mathrm{b}^{(i)} \right). \tag{18}$$

Substitute Eq. (17) into Eq. (18), then the effective bulk modulus is expressed as

$$\kappa_\mathrm{eff} = \frac{(\kappa_1 - \kappa_2)\,\bar{\varepsilon}_\mathrm{b}(r_1) + (\kappa_2 - \kappa_3)\,\bar{\varepsilon}_\mathrm{b}(r_2) + \kappa_3 \bar{\varepsilon}_\mathrm{b}(r_3)}{\bar{\varepsilon}_\mathrm{b}(r_3)}. \tag{19}$$

Integrate Eq. (14b) on a circular region S of the radius R. The deviatoric part $\bar{\boldsymbol{\varepsilon}}'$ of the overall strain $\bar{\boldsymbol{\varepsilon}} = \int \boldsymbol{\varepsilon} \mathrm{d}S$ can be expressed as

$$\bar{\boldsymbol{\varepsilon}}' = \bar{e} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{20}$$

where

$$\bar{e}(R) = R \sum_n \left[u_{r,n}(R)\, p_n - u_{\theta,n}(R)\, q_n \right], \tag{21}$$

with

$$p_n = \int_0^{2\pi} \cos n\theta \cos 2\theta \mathrm{d}\theta, \tag{22a}$$

$$q_n = \int_0^{2\pi} \sin n\theta \sin 2\theta \mathrm{d}\theta. \tag{22b}$$

Because of the orthogonal properties of the trigonometric functions, the nonvanishing coefficients in Eqs. (22a,22b) are $p_2 = q_2 = \pi$. The overall deviatoric stress $\bar{\tau}$ is related to the deviatoric strain $\bar{e}$ by shear modulus, $\bar{\tau} = 2\mu\bar{e}$. Therefore, the deviatoric stress in the ith region can be computed by

$$\bar{\tau}^{(i)} = 2\mu_i \bar{e}^{(i)}. \tag{23}$$

The effective shear modulus can be defined as

$$\mu_\mathrm{eff} = \sum_i \bar{\tau}^{(i)} \bigg/ \left(2 \sum_i \bar{e}^{(i)} \right). \tag{24}$$

Substituting Eq. (23) into Eq. (24), the effective shear modulus of the three-phase composites is

$$\mu_\mathrm{eff} = \frac{(\mu_1 - \mu_2)\,\bar{e}(r_1) + (\mu_2 - \mu_3)\,\bar{e}(r_2) + \mu_3 \bar{e}(r_3)}{\bar{e}(r_3)}. \tag{25}$$

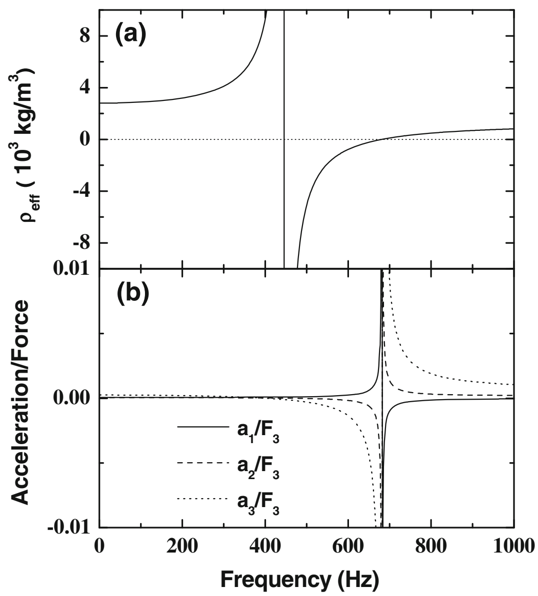

Fig. 2 **a** Effective mass density of a rubber-coated lead cylinders in an epoxy matrix; **b** averaged acceleration of the particle, coating and matrix versus the external force

5 Numerical examples

Numerical examples for predicting effective dynamic properties of 2D acoustic metamaterials will be given in this section. First, consider the composite consisting of rubber-coated lead cylinders embedded in an epoxy matrix. The material parameters are $\rho_1 = 11,600\,\text{kg/m}^3$, $\lambda_1 = 42.3\,\text{GPa}$, $\mu_1 = 14.9\,\text{GPa}$ for lead, $\rho_2 = 1,300\,\text{kg/m}^3$, $\lambda_2 = 0.6\,\text{MPa}$, $\mu_2 = 0.04\,\text{MPa}$ for silicone rubber, and $\rho_3 = 1,180\,\text{kg/m}^3$, $\lambda_3 = 4.43\,\text{GPa}$, $\mu_3 = 1.59\,\text{GPa}$ for epoxy. The radius of the inner cylinder is 5.0 mm, the coating thickness is 2.0 mm and the volume fraction of the coated cylinder is 30 %. Figure 2a shows effective mass density of the composite predicted by Eq. (12). It can be found that negative effective mass occurs in a narrow band above the resonant frequency 445 Hz. To discover the mechanism of the negative mass effect, the averaged acceleration of the lead particle a_1, rubber coating a_2 and epoxy matrix a_3 versus the external force F_3 is shown in Fig. 2b. It can be seen that around the frequency 445 Hz, the lead cylinder resonates and applies an out-of-phase force on the coating and matrix, so that the averaged accelerations of the coating and matrix become negative. Above this frequency, negative acceleration/force relation overwhelms the system due to the resonant effect, resulting in negative effective mass. The result allows us to construct a mass-spring structure equivalent to the composite unit, as shown in Fig. 3a, where the mass m_1, spring G and mass m_0 represent, respectively, the lead particle, rubber coating and epoxy matrix. The effective mass of the mass-spring structure is derived to be [10]

$$m_{\text{eff}} = m_0 \left(1 + \frac{\omega_c^2}{\omega_0^2 - \omega^2} \right), \tag{26}$$

where $\omega_c = \sqrt{G/m_0}$ and $\omega_0 = \sqrt{G/m_1}$. From Eq. (26), the inner mass resonates at ω_0 and moves out of phase with respect to the outer mass, so that effective mass becomes negative. Therefore, negative mass phenomenon of the analyzed composite is due to the resonant effect of the lead particle.

Further consider the case that the lead particle is fixed. The equivalent mass-spring structure is shown in Fig. 3b. Suppose that the mass m_0 has a displacement u_0 under a harmonic force F of angular frequency ω. Newton's law of motion gives the relation $F = -m_0\omega^2 u_0 + G u_0$. If the structure is considered as a homogeneous structure defined by an effective mass m_{eff}, we have $F = -m_{\text{eff}}\omega^2 u_0$. The effective mass m_{eff} is given by [3]

$$m_{\text{eff}} = m_0 \left(1 - \frac{\omega_c^2}{\omega^2} \right). \tag{27}$$

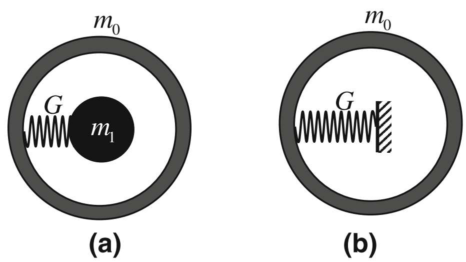

Fig. 3 The mass-spring structure with the free inner mass (**a**) and fixed inner mass (**b**)

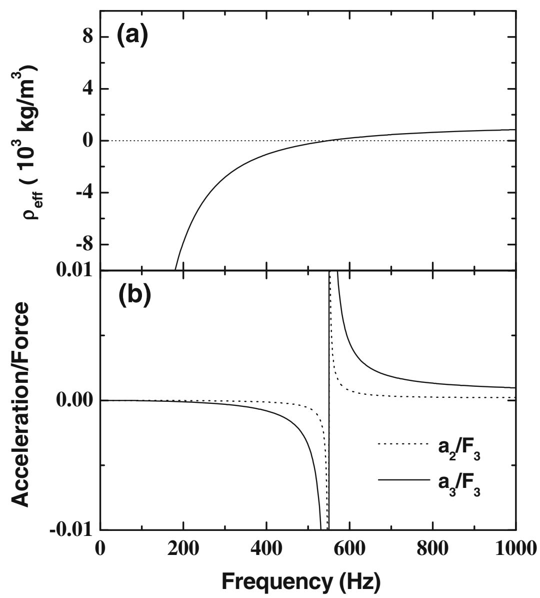

Fig. 4 **a** Effective mass density of a rubber-coated lead cylinders in an epoxy matrix, where the lead cylinder is fixed; **b** averaged acceleration of the coating and matrix versus the external force

According to Eq. (27), the effective mass is negative below the cutoff frequency ω_c. For the composite analyzed in Fig. 2, the effective mass density predicted by the proposed model is shown in Fig. 4a, when the lead particle is fixed. It can be found that effective mass is negative below the frequency 550 Hz. We plot the averaged acceleration a_2 and a_3 of the rubber coating and epoxy matrix versus the external force F_3 in Fig. 4b. It is seen that the accelerations of the coating and matrix respond out of phase with respect to the applied force, as the result of negative effective mass below a cutoff frequency. The proposed model clearly demonstrates the result discovered from the mass-spring model.

Consider the composite made of bubble-contained-water cylinders embedded in an epoxy matrix. Material parameters are $\rho_1 = 1.23\,\text{kg/m}^3$, $\lambda_1 = 0.142\,\text{MPa}$ for air and $\rho_2 = 1{,}000\,\text{kg/m}^3$, $\lambda_2 = 2.22\,\text{GPa}$ for water. The radius of the air cylinder is 2.0 mm, the coating thickness is 78.0 mm and the volume fraction of the coated cylinder is 10 %. It has been demonstrated that such a kind of composite could realize an negative effective bulk modulus based on the monopolar resonance of the inclusions [15,17]. Figure 5a, b show, respectively, the effective bulk modulus κ_{eff} of the composite and the averaged bulk strain $\varepsilon_i = \bar{\varepsilon}_{\text{b}}^{(i)}$ of each phase versus the total bulk strain $\varepsilon_t = \sum \bar{\varepsilon}_{\text{b}}^{(i)}$. In Fig. 5a, a negative effective bulk modulus can be observed at frequencies ranging from 1,520 to 1,770 Hz. The underlying mechanism can be understood from Fig. 5b. Assume the composite cylinder is in the state of expansion $\varepsilon_t > 0$, then the water coating can be greatly compressed due to the resonant effect. Since the modulus of water is much greater than that of air, the composite is under compressive stress, as the result of a negative effective bulk modulus.

As final example, consider rubber-coated epoxy cylinders embedded in the polyethylene foam HD115. Material parameters of the polyethylene foam are $\rho_3 = 115\,\text{kg/m}^3$, $\lambda_3 = 6\,\text{MPa}$, $\mu_3 = 3\,\text{MPa}$ [15]. The radius of the epoxy cylinder is 5 mm, the coating thickness is 7 mm and the filling fraction of the coated cylinder

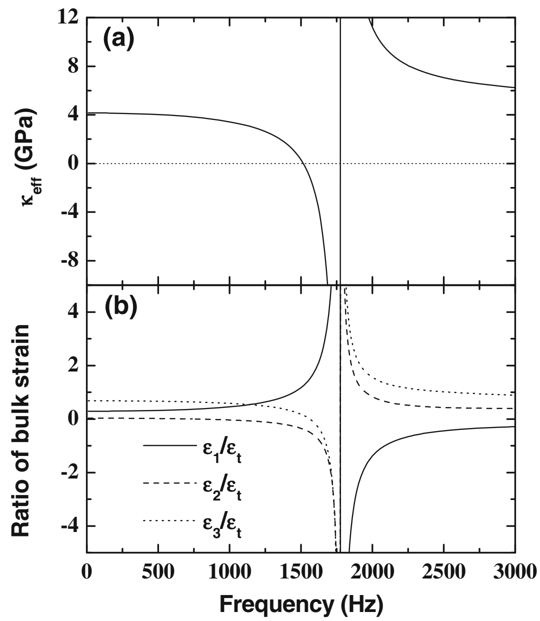

Fig. 5 **a** Effective bulk modulus κ_{eff} of the composite with bubble-contained-water cylinders embedded in an epoxy matrix; **b** averaged bulk strain ε_i of each phase versus the total bulk strain ε_t

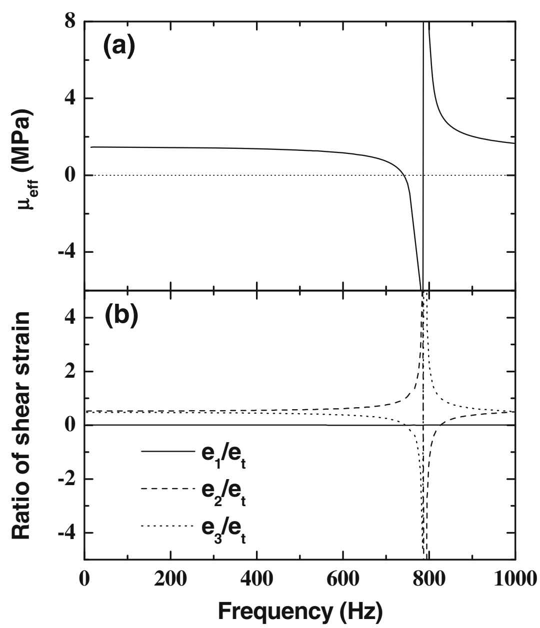

Fig. 6 **a** Effective shear modulus μ_{eff} of the composite composed of rubber-coated epoxy cylinders embedded in the polyethylene foam; **b** averaged shear strain e_i of each phase versus the total shear strain e_t

is 30 %. The effective shear modulus μ_{eff} of the composite and the averaged shear strain $\mathrm{e}_i = \bar{e}^{(i)}$ of each phases versus the total shear strain $\mathrm{e}_t = \sum \bar{e}^{(i)}$ are shown in Fig. 6a, b, respectively. It is seen that the proposed model predicts negative effective shear modulus in the frequency band around 780 Hz. With help of Fig. 6b, the mechanism of negative effective shear modulus can be analyzed. The shear deformation of the inner core is trivial at all frequencies. A resonant effect takes place at around 780 Hz, so that the foam matrix undergoes the negative shear strain when the total shear strain is positive. Since the polyethylene foam is stiffer than the soft rubber, negative shear stress dominates the whole composite. Negative effective shear modulus is defined for composites with positive shear strain under the loading of negative shear stress.

In a previous paper [15], the physical mechanisms of negative effective mass, bulk modulus and shear modulus have been explored based on the proposed model for the case of spherical inclusions. Here, we have demonstrated that similar mechanisms can be observed in 2D acoustic metamaterials with cylindrical inclusions. The proposed method can predict negative effective parameters of 2D acoustic metamaterials and disclose the physical mechanism of anomalous properties.

6 Conclusion

In conclusion, we propose dynamic effective model for predicting negative effective mass, bulk and shear moduli of metamaterials with cylindrical inclusions. The model derives the analytic expressions for effective material parameters based on the averaged displacement and stress fields. Numerical examples demonstrate that the out-of-phase resonant response of inclusions is the origin of negative effective mass, bulk and shear modulus. In addition, the proposed model can also predict effective properties of composites with negative effective mass below a cutoff frequency. The proposed method will be helpful for the design of acoustic metamaterials with resonant microstructures.

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