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Effective in plane moduli of composites with a micropolar matrix and coated fibers

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Abstract

Stress and couple stress distributions are determined analytically for a coated fiber in an infinite micropolar matrix, loaded remotely by classical symmetric stresses. The determined stresses are then compared with those predicted by Cauchy theory, a size dependence is predicted in the framework of micropolar theory, and when the fiber's diameter is much larger than the characteristic length of the matrix, the classical prediction can be recovered. The exact average stress in a fiber for a fiber-matrix system (without interphase) is also compared with that approximately derived by the average equivalent inclusion method (AEIM), based on the micropolar Eshelby tensor given by Cheng and He [Int. J. Eng. Sci. 35 (7) (1997) 659], the results show that the approximate AEIM method can give an accurate prediction for the size dependence of the average stress in a fiber. A micro–macro transition method is also proposed to determine effective in plane moduli of a heterogeneous micropolar material, the effective shear and in plane bulk moduli of a micropolar composite with coated fibers are derived analytically by extending Mori–Tanaka's method to a micropolar composite, and the effective shear and in plane bulk moduli for a two-phase fiber composite (without interphase) are also obtained as a special case. The results show that the effective in plane bulk modulus is identical to that obtained by Cauchy theory, however the effective shear modulus depends on fiber's diameter, it increases with decreasing fiber's diameter at the same fiber volume fraction, and the classical prediction is recovered when the fiber's diameter becomes large compared to the matrix characteristic length.

Keywords: Micropolar; Micromechanics; Coated fiber; Effective moduli; Composite

1. Introduction

In this paper, we are interested in the localization problem for a coated fiber in an infinite micropolar matrix, and the effective in plane moduli of such composite, both the fiber and the interphase can be micropolar materials. The motivation of this work comes one part directly from the coating technology to improve the strength and toughness of composite materials (Verghese, 1999; Gao and Mader, 2002),

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another part from the need to establish a rigorous micromechanical method to explain the well-observed size effect of composite materials (Lloyd, 1994; Yang et al., 1990). For Cauchy composites, there are different models to predict effective properties of composites from their microscale parameters (see for example Mura, 1982; Nemat-Nasser and Hori, 1993; Torquato, 2002; Milton, 2002). The developed analytical methods are usually based on the Eshelby's fundamental solution, or on the solutions for a coated or multilayer sphere or cylinder (generalized self-consistent method for example). Although a great success has been achieved, the micromechanical methods developed in the framework of Cauchy theory fail to predict a size dependence for overall properties, especially for the overall plasticity of composites (Lloyd, 1994; Yang et al., 1990; Liu and Hu, in press).

Any material is basically heterogeneous in nature, homogenization of the material depends on length scales in which we are interested. Consider a fundamental problem for a composite material, an inhomogeneity is placed into a matrix material, as shown in Fig. 1. The matrix material has its own characteristic length due to its inner microstructure, for example, the grain size for a polycrystal material. When the size of the inhomogeneity is much larger than the characteristic length of the matrix (Fig. 1a), in this case, the matrix material can be homogenized as a Cauchy material. However, as shown in Fig. 1b, when the size of the reinforced phase is comparable to the characteristic length of the matrix, nonlocal nature of the matrix material may become important. This problem is relevant for metal matrix composites, nanocomposites. It is widely accepted that to include this nonlocal nature of a material in a homogenized continue formulation, high order continuum theories with inherent length scales must be assigned for the matrix material. Different high order theories and methods have been advanced in the literature, as early as in 1909, Cosserat brothers proposed a continuum theory by introducing, in addition to a displacement vector, a set of directors at each material point to characterize the kinetic motion of microstructures inside of this point. This idea has been further elaborated by Mindlin and Tiersten (1962), Toupin (1962), and especially by Eringen (1968), the detailed results have been recently summarized in the monograph by Eringen (1999). In micropolar theory, a rotation vector is introduced at each material point as an additional degree of freedom, and this gives rise to a couple stress, conjugated with the gradient of the rotation vector, defined as a torsion. Due to dimensional consistency, the relation between the couple stress and the torsion brings naturally some length scales, usually defined as material characteristic lengths. When the micro-rotation in micropolar theory is identified as the rigid rotation of a material point, this leads to so-called couple stress theory, as discussed by Mindlin and Tiersten (1962), and recently reformulated and extended to plasticity by Fleck and Hutchinson (1997) for exploring the size effect due to heterogeneous plastic deformations. Some new developments along this line can be found in the references Gao et al. (1999) and Hwang et al. (2002). Another method for explaining the size effect from a continue point of view has been proposed and developed by Aifantis (1984, 1987), he introduced only a high order displacement gradient in a constitutive relation.

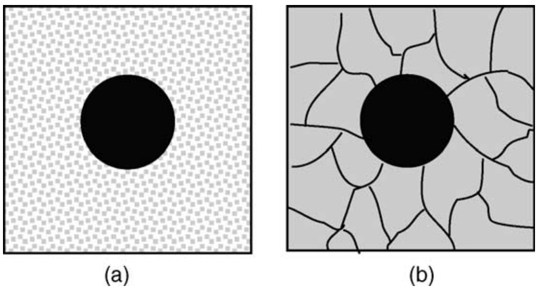

Fig. 1. Length scale conditions (a) the size of fiber is much larger than the characteristic length of the matrix; (b) the size of fiber is comparable to the characteristic length of the matrix.

Although there are lots of works devoted to the theories of a high order continuum, systematic analyses on effective properties for a heterogeneous micropolar material are few. There are two major problems concerning on the evaluation of overall properties for a micropolar composite: the first is a proper definition of a micro–macro transition principle; the second is the solution of some typical localization problems. For the first problem, Forest et al. (1999, 2000) have examined different boundary conditions on a representative volume element (RVE), and they proposed corresponding micro–macro transition methods; and for the second problem, following the idea of Eshelby, Cheng and He (1995, 1997) derived the analytical solutions for a homogeneous micropolar material in which a cylindrical or a spherical region is subjected to a uniform eigenstrain and a uniform eigentorsion. Their pioneer work makes the equivalent inclusion method possible for a micropolar composite, and we will come back to this point in the following section. For the other interesting works concerning the localization problem, the readers can refer to the monograph by Eringen (1999). Recently Sharma and Dasgupta (2002), Liu and Hu (in press) extended Mori–Tanaka's method to evaluate the elastic moduli for a two-phase micropolar composite. The other works concerning on the effective property for a micropolar composite are usually numerical in nature, for example, Yuan and Tomita (2001) proposed a numerical method (Finite Element method) to evaluate the effective moduli for periodic voids or fibers in a micropolar matrix; and the effective elastoplastic property for a micropolar polycrystal material is also examined by Forest et al. (2000) through a finite element method. Finally a shift property for the effective planar moduli of a micropolar composite was examined by Ostoja-Starzewski and Jasiuk (1995).

The objective of this paper is to propose an analytical method for estimating effective moduli of a micropolar composite with coated fibers. The paper will be arranged as follows: the detailed solution for a coated fiber in a micropolar matrix will be presented in Section 2, the influence of interphase properties on stress concentration is discussed in Section 3; an approximate average equivalent inclusion method (AEIM) based on the micropolar Eshelby tensor is utilized to determine the average stress in a fiber, and it is also compared with the exact result, this will be presented in Section 4. In Section 5, an analytical method for evaluating effective in plane moduli of a micropolar composite will be presented.

2. Theoretical formulation

2.1. Preliminary

We consider the following problem: a coated long fiber is embedded into an infinite matrix under a remote uniform classical stress, as shown in Fig. 2. The materials of these three phases are centro-symmetric and isotropic micropolar materials. A cylindrical coordinate system (r, θ, z) is established with z-axis as the longitudinal direction and r–θ plan lays on the transverse plan (Fig. 2). The fiber has a radius R_1, and the coated layer has a radius R_2.

In absence of body force and body moment, the governing equations for determining elastic stress and couple stress are given by the following three sets of equations (Eringen, 1999) with a proper boundary condition:

Kinetic relation

$$\varepsilon_{ij} = u_{j,i} - e_{kij}\phi_k, \tag{1a}$$

$$k_{ij} = \phi_{j,i}. \tag{1b}$$

Balance equations of momentum and moment momentum

$$\sigma_{ij,i} = 0, \tag{2a}$$

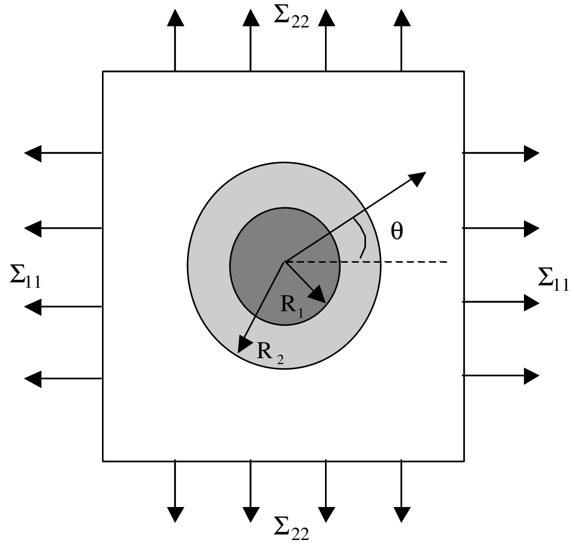

Fig. 2. Sketch of a three-phase model.

$$m_{ij,i} + e_{jik}\sigma_{ik} = 0. \tag{2b}$$

A centro-symmetric and isotropic and constitutive relation

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + (\mu+\kappa)\varepsilon_{ij} + (\mu-\kappa)\varepsilon_{ji}, \tag{3a}$$

$$m_{ij} = \alpha k_{kk}\delta_{ij} + \beta k_{ij} + \gamma k_{ji}. \tag{3b}$$

where u_j is a displacement vector and ϕ_j is a micro-rotation vector. σ_{ij}, ε_{ij}, k_{ij} and m_{ij} are respectively stress, strain, torsion and couple stress tensors, generally, they are asymmetric. δ_{ij} is the Kronecker delta, e_{ijk} is the permutation tensor. λ, μ are the classical Lame constants in elasticity, κ, α, β, γ are the additional elastic constants introduced in micropolar theory. The constants μ, λ, κ have a dimension of force per unit area, and γ, β, α have a dimension of force. The following relations for the Young's modulus, Poisson's ratio and bulk modulus E, v and k still hold

$$E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}, \quad v = \frac{\lambda}{2(\lambda+\mu)}, \quad k = \lambda + \frac{2}{3}\mu. \tag{4}$$

These constants relate the symmetric parts of stress and strain just as a Cauchy material (Nowacki, 1986).

For the first kind problem of a plane strain condition considered in this paper, the above equations become

$$\varepsilon_{\alpha\beta} = u_{\beta,\alpha} - e_{3\alpha\beta}\phi_3, \quad k_{\alpha 3} = \phi_{3,\alpha}, \tag{5a}$$

$$\sigma_{\alpha\beta,\alpha} = 0, \quad m_{\beta 3,\beta} + e_{3\alpha\beta}\sigma_{\alpha\beta} = 0, \tag{5b}$$

$$\sigma_{\alpha\beta} = \lambda\varepsilon_{\zeta\zeta}\delta_{\alpha\beta} + (\mu+\kappa)\varepsilon_{\alpha\beta} + (\mu-\kappa)\varepsilon_{\beta\alpha}, \quad m_{\alpha 3} = \beta k_{\alpha 3} = \beta\phi_{3,\alpha}. \tag{5c}$$

The subscript in Greek letter ranges from 1 to 2. The relation (4) still holds except that the bulk modulus in this case becomes in plane bulk modulus, and it is defined by $k = \lambda + \mu$. The Young's modulus and Poisson's ratio should be interpreted as the in plane quantities.

2.2. Solution of the problem

For the considered problem, all the variables are independent of z, and they are only functions of r, θ. According to Eringen (1999), the general solution can be obtained by introducing in each region the stress and couple stress potentials F_i and G_i, ($i = 1, 2, 3$, it refers to the fiber, interphase and matrix respectively), and the stress and couple stress are related by the potentials in a cylindrical coordinate system as

$$\sigma^i_{rr} = \frac{1}{r}\frac{\partial F_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F_i}{\partial \theta^2} - \frac{1}{r}\frac{\partial^2 G_i}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial G_i}{\partial \theta}, \tag{6a}$$

$$\sigma^i_{\theta\theta} = \frac{\partial^2 F_i}{\partial r^2} + \frac{1}{r}\frac{\partial^2 G_i}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial G_i}{\partial \theta}, \tag{6b}$$

$$\sigma^i_{r\theta} = -\frac{1}{r}\frac{\partial^2 F_i}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial F_i}{\partial \theta} - \frac{1}{r}\frac{\partial G_i}{\partial r} - \frac{1}{r^2}\frac{\partial^2 G_i}{\partial \theta^2}, \tag{6c}$$

$$\sigma^i_{\theta r} = -\frac{1}{r}\frac{\partial^2 F_i}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial F_i}{\partial \theta} + \frac{\partial^2 G_i}{\partial r^2}, \tag{6d}$$

$$m^i_{rz} = \frac{\partial G_i}{\partial r}, \tag{6e}$$

$$m^i_{\theta z} = \frac{1}{r}\frac{\partial G_i}{\partial \theta}. \tag{6f}$$

The compatibility conditions for each region now become (Eringen, 1999)

$$\frac{\partial}{\partial r}(G_i - c_i^2 \nabla^2 G_i) = -2(1 - \nu_i) b_i^2 \frac{1}{r}\frac{\partial}{\partial \theta}(\nabla^2 F_i), \tag{7a}$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}(G_i - c_i^2 \nabla^2 G_i) = 2(1 - \nu_i) b_i^2 \frac{\partial}{\partial r}(\nabla^2 F_i), \tag{7b}$$

where

$$b_i^2 = \frac{\beta_i}{4\mu_i} = \frac{\kappa_i}{\kappa_i + \mu_i} c_i^2 = d_i c_i^2, \quad \nu_i = \frac{\lambda_i}{2(\lambda_i + \mu_i)}, \tag{8}$$

where $d_i = \kappa_i/(\kappa_i + \mu_i)$, $c_i^2 = \beta_i(\kappa_i + \mu_i)/4\mu_i\kappa_i$. Constants b_i, c_i have a dimension of length, they can be considered as characteristic lengths of a micropolar material.

Eqs. (7) lead to the following differential equations for the stress and couple stress potentials

$$\nabla^4 F_i = 0, \tag{9a}$$

$$\nabla^2 (G_i - c_i^2 \nabla^2 G_i) = 0, \tag{9b}$$

where ∇^2 is Laplacian operator.

The general solutions of equations (9) are obtained for the region i as

$$F_i = A^i_1 R_1^2 \operatorname{Log} r + A^i_2 r^2 + (A^i_3 R_1^2 + A^i_4 r^2 + A^i_5 R_1^4 r^{-2} + A^i_6 R_1^{-2} r^4)\cos 2\theta, \tag{10a}$$

$$G_i = [A^i_7 R_1^4 r^{-2} + A^i_8 r^2 + A^i_9 R_1^2 K_2(r/c_i) + A^i_{10} R_1^2 I_2(r/c_i)]\sin 2\theta, \tag{10b}$$

where $I_M(r/c_i)$ is the modified Bessel function of the first kind of order M, $K_M(r/c_i)$ is the modified Bessel function of the second kind of order M, and A_j^i (the superscript $i = 1, 2, 3$, referring to the different regions, and $j = 1, 2, \ldots, 10$) are the constants to be determined.

It is found that Eqs. (7) are satisfied by setting

$$A_7^i = 8b_i^2(1-\nu_i)R_1^{-2}A_3^i, \tag{11a}$$

$$A_8^i = 24b_i^2(1-\nu_i)R_1^{-2}A_6^i. \tag{11b}$$

The local stress, couple stress, displacement and micro-rotation fields are derived with the help of Eqs. (6) and (10) and the constitutive relations for each phase, they are for the region i:

$$\sigma_{rr}^i = A_1^iR_1^2r^{-2} + 2A_2^i - 2\{2A_3^iR_1^2r^{-2} + (A_4^i + A_8^i) + 3(A_5^i - A_7^i)R_1^2r^{-4} + A_9^iR_1^2[3r^{-2}K_0(r/c_i) + (6r^{-3}c_i + c_i^{-1}r^{-1})K_1(r/c_i)] + A_{10}^iR_1^2[3c_ir^{-2}I_0(r/c_i) - (6r^{-3}c_i + c_i^{-1}r^{-1})I_1(r/c_i)]\}\cos 2\theta, \tag{12a}$$

$$\sigma_{\theta\theta}^i = -A_1^iR_1^2r^{-2} + 2A_2^i + 2\{(A_4^i + A_8^i) + 3(A_5^i - A_7^i)R_1^4r^{-4} + 6A_6^iR_1^{-2}r^2 - 2A_9^iR_1^2[3r^{-2}K_0(r/c_i) + (6r^{-3}c_i + c_i^{-1}r^{-1})K_1(r/c_i)] - 2A_{10}^iR_1^2[3r^{-2}I_0(r/c_i) - (6r^{-3}c_i + c_i^{-1}r^{-1})I_1(r/c_i)]\}\cos 2\theta, \tag{12b}$$

$$\sigma_{r\theta}^i = \{-2A_3^iR_1^2r^{-2} + 2(A_4^i + A_8^i) - 6(A_5^i - A_7^i)R_1^4r^{-4} + 6A_6^iR_1^{-2}r^2 + A_9^iR_1^2[6r^{-2}K_0(r/c_i) + (12c_iR_1r^{-3} + c_i^{-1}r^{-1})K_1(r/c_i)] + A_{10}^iR_1^2[6r^{-2}I_0(r/c_i) - (12c_iR_1r^{-3} + c_i^{-1}r^{-1})I_1(r/c_i)]\}\sin 2\theta, \tag{12c}$$

$$\sigma_{\theta r}^i = \{-2A_3^iR_1^2r^{-2} + 2(A_4^i + A_8^i) - 6(A_5^i - A_7^i)R_1^4r^{-4} + 6A_6^iR_1^{-2}r^2 + A_9^iR_1^2[(6r^{-2} + c^{-2})K_0(r/c_i) + (12r^{-3}c_i + 3r^{-1}c_i^{-1})K_1(r/c_i)] + A_{10}^iR_1^2[(6r^{-2} + c^{-2})I_0(r/c_i) - (12r^{-3}c_i + 3r^{-1}c_i^{-1})I_1(r/c_i)]\}\sin 2\theta, \tag{12d}$$

$$m_{rz}^i = \{-2A_7^iR_1^4r^{-3} + 2A_8^ir - A_9^iR_1^2[2r^{-1}K_0(r/c_i) + (4r^{-2}c_i + c_i^{-1})K_1(r/c_i)] - A_{10}^iR_1^2[2r^{-1}I_0(r/c_i) - (4r^{-2}c_i + c_i^{-1})I_1(r/c_i)]\}\sin 2\theta, \tag{12e}$$

$$m_{\theta z}^i = 2\{A_7^iR_1^4r^{-3} + A_8^ir + A_9^iR_1^2[K_0(r/c_i)r^{-1} + 2r^{-2}c_iK_1(r/c_i)] + A_{10}^iR_1^2[I_0(r/c_i)r^{-1} - 2r^{-2}c_iI_1(r/c_i)]\}\cos 2\theta. \tag{12f}$$

The displacement and micro-rotation fields are

$$u_r^i = -\frac{R_1^2}{2r\mu_i}A_1^i + \frac{r}{\lambda_i + \mu_i}A_2^i + \frac{\cos 2\theta}{\mu_i}\left\{\frac{R_1^2(\lambda_i + 2\mu_i)}{\lambda_i + \mu_i}r^{-1}A_3^i - r(A_4^i + A_8^i) + r^{-3}R_1^4(A_5^i - A_7^i) - \frac{2\nu_i}{R_1^2}r^3A_6^i + [rK_0(r/c_i) + 2c_iK_1(r/c_i)]R_1^2r^{-2}A_9^i - [rI_0(r/c_i) - 2c_iI_1(r/c_i)]R_1^2r^{-2}A_{10}^i\right\}, \tag{13a}$$

$$u_\theta^i = C_ir + \frac{\sin 2\theta}{2\mu_i}\left\{-\frac{2R_1^2\mu_i}{\lambda_i + \mu_i}r^{-1}A_3^i + 2r(A_4^i + A_8^i) + 2r^{-3}R_1^4(A_5^i - A_7^i) - \frac{2(2\lambda_i + 3\mu_i)}{\lambda_i + \mu_i}R_1^{-2}r^3A_6^i + [2r^{-1}K_0(r/c_i) + (4c_ir^{-1} + 1/c_i)K_1(r/c_i)]R_1^2A_9^i + [-2r^{-1}I_0(r/c_i) + (4c_ir^{-1} + 1/c_i)I_1(r/c_i)]R_1^2A_{10}^i\right\}, \tag{13b}$$

$$\phi_z^i = \frac{\sin 2\theta}{r^2\beta_i}\{R_1^4A_7^i + r^4A_8^i + rR_1^2[rK_0(r/c_i) + 2c_iK_1(r/c_i)]A_9^i + [rI_0(r/c_i) - 2c_iI_1(r/c_i)]A_{10}^i\}. \tag{13c}$$

The remote boundary condition can be written as $r \to \infty$, $\sigma_{rr}^3 = \frac{1}{2}(\Sigma_{11} + \Sigma_{22}) + \frac{1}{2}(\Sigma_{11} - \Sigma_{22})\cos 2\theta$, $\sigma_{r\theta}^3 = \frac{1}{2}(\Sigma_{22} - \Sigma_{11})\sin 2\theta$, $m_{rz}^3 = 0$.

These provide the following conditions ($R_2 \leqslant r < \infty$)

$$A_6^3 = A_8^3 = A_{10}^3 = 0, \quad A_2^3 = \frac{1}{2}(\Sigma_{11} + \Sigma_{22}), \quad A_4^3 = \frac{1}{2}(\Sigma_{11} - \Sigma_{22}). \tag{14}$$

For the fiber ($0 \leqslant r \leqslant R_1$), due to the condition of the finite stress and couple stress, we have

$$A_1^1 = A_3^1 = A_5^1 = A_7^1 = A_9^1 = 0. \tag{15}$$

The other unknown constants can be determined from the continuity condition at the interface between the fiber and interphase, and the interface between the interphase and matrix, respectively, which are written as (for $i = 1, 2$)

$$\begin{aligned} &u_r^i(R_i) = u_r^{i+1}(R_i), \quad u_\theta^i(R_i) = u_\theta^{i+1}(R_i), \quad \sigma_{rr}^i(R_i) = \sigma_{rr}^{i+1}(R_i), \\ &\sigma_{r\theta}^i(R_i) = \sigma_{r\theta}^{i+1}(R_i)m_{rz}^i(R_i) = m_{rz}^{i+1}(R_i), \quad \phi_z^i(R_i) = \phi_z^{i+1}(R_i). \end{aligned} \tag{16}$$

The above condition can be rewritten in a more compact form, if we note

$$\overline{A}^1 = \{0, A_2^1, 0, A_4^1, 0, A_6^1, 0, A_8^1, 0, A_{10}^1\}^T, \tag{17a}$$

$$\overline{A}^2 = \{A_1^2, A_2^2, A_3^2, A_4^2, A_5^2, A_6^2, A_7^2, A_8^2, A_9^2, A_{10}^2\}^T, \tag{17b}$$

$$\overline{A}^3 = \{A_1^3, A_2^3, A_3^3, A_4^3, A_5^3, 0, A_7^3, 0, A_9^3, 0\}^T. \tag{17c}$$

With the help of the detailed expressions for the local fields (Eqs. (12) and (13)) and the condition (11), the condition (16) can be written together in a compact form as ($i = 1, 2$)

$$M_i(R_i)\overline{A}^i = M_{i+1}(R_i)\overline{A}^{i+1}. \tag{18}$$

The expression for the matrix $M_i(r)$ is given in Appendix A. Until now, the unknown constants are determined completely, so as to the local stress and couple stress fields.

3. Influence of a micropolar interphase on local stress distribution

3.1. A micropolar interphase with a classical matrix and a classical fiber

For a Cauchy material, there are many works devoted to understanding stress transfer mechanisms by a presence of an interphase (Jasiuk and Kouider, 1993; Huang and Rokhlin, 1996; Xun et al., in press), it is found that there are large stress gradients in this thin interphase layer. To model the response of this interphase layer usually with a complex heterogeneous microstructure and under a large stress gradient, we propose to consider it as a micropolar material, and the other phases are classical Cauchy materials. Only a remote uniaxial load $\Sigma_{11} \neq 0$ is examined, the material constants used are $\mu_1{:}\mu_2{:}\mu_3 = 100{:}30{:}1$, $v_1 = 0.25$, $v_2 = 0.35$, $v_3 = 0.4$, $d_2 = 0.4$ and $R_2 = 1.1R_1$.

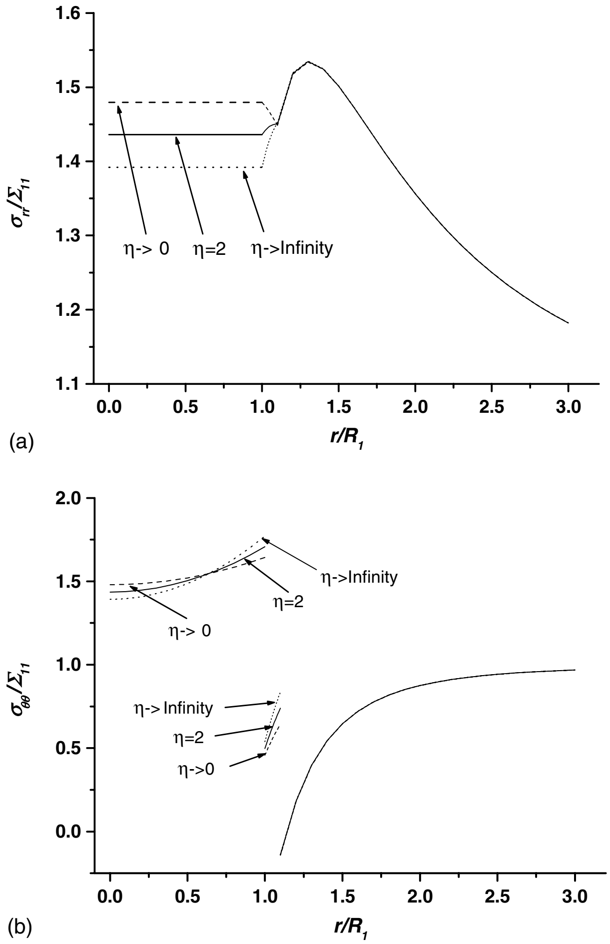

Fig. 3. Variation of stresses for different values of parameter $\eta = R_1/c_2$: (a) σ_{rr}/Σ_{11} along $\theta = 0$; (b) $\sigma_{\theta\theta}/\Sigma_{11}$ along $\theta = \pi/2$.

The variation of stress component σ_{rr}/Σ_{11} along $\theta = 0$ due to the presence of an interphase is shown in Fig. 3a for different parameters $\eta = R_1/c_2$, the variation of stress component $\sigma_{\theta\theta}/\Sigma_{11}$ along $\theta = \pi/2$ is also illustrated in Fig. 3b. In the above computations, $\eta \to \infty$ corresponds to a classical interphase (a Cauchy material). It is found that the micropolar effect of this thin interphase layer has an important influence on the stress distribution in the interphase layer and in the fiber. For example, the fiber with a small diameter supports more average radial stress along $\theta = 0$ compared to the large one, as indicated in Fig. 3a.

3.2. A micropolar interphase with a micropolar matrix and a micropolar fiber

In this section, the stress concentration factor for a coated fiber in an infinite matrix under a uniaxial load will be estimated, we are only illustrated the cases where the fiber is rigid or void, and the interphase and the matrix are micropolar materials. The material constants used in the computation are: $v_2 = 0.25$, $v_3 = 0.35$, $d_2 = 0.3$, $d_3 = 0.2$, $R_2 = 1.1R_1$ and $c_2 = c_3$. The ratio of the shear modulus of the interphase to that of the matrix will be specified when necessary. We have checked that when the interphase layer vanishes or takes the same material constants as those for the fiber or the matrix, the results given by Weitsman (1966) can be recovered.

Firstly, we examine the case of a rigid fiber and analyze the influence of interphase properties on the stress concentration factor, defined by $K = \sigma_{\theta\theta}/\Sigma_{11}$ along the line $\theta = \pi/2$ at $r = R_1$ from the interphase. The variations of the stress concentration factor as a function of parameter $\eta = R_1/c_2$ are shown in Fig. 4 for $\mu_2{:}\mu_3 = 13.0$ and $\mu_2{:}\mu_3 = 0.13$, corresponding roughly to a hard and soft interphase respectively. The stress concentration factors for the corresponding Cauchy material are also included for a comparison. It is found that when the fiber's diameter is large (large $\eta = R_1/c_2$ value), the prediction based on micropolar theory coincides with that predicted by Cauchy theory, and the size dependence of the concentration factor on the fiber's diameter can be neglected for the both hard and soft interphases. However when the fiber's diameter approaches to the characteristic length of the interphase (c_2), the prediction on the stress concentration factor by micropolar theory deviates significantly from that predicted by the classical method.

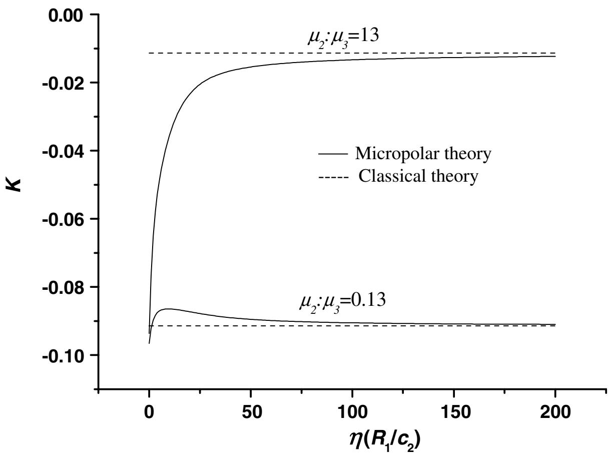

Fig. 4. Influence of fiber's diameter on stress concentration factor for different values of $\mu_2{:}\mu_3$.

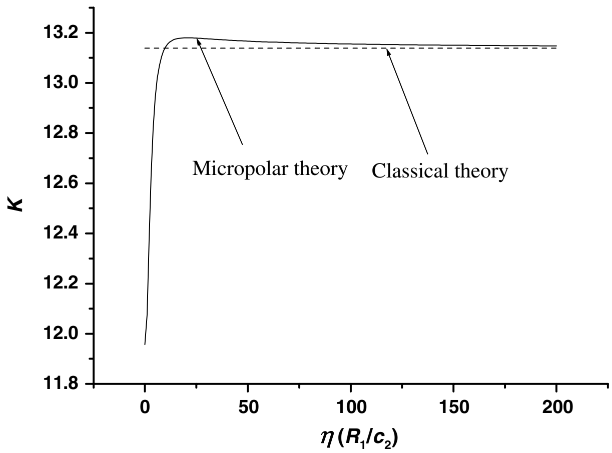

Fig. 5. Influence of void's size on the stress concentration factor for $\mu_2{:}\mu_3 = 13.0$.

The stress concentration due to a coated hole is also examined, and the results are shown in Fig. 5. It is seen that the region dominated by void size is sharply reduced compared to the case of a rigid fiber. When the void size approaches to the characteristic length of the interphase (c_2), the stress concentration factor is significantly reduced for the examined hard interphase. It is also found that for a void with a hard interphase, the stress concentration factors estimated by both micropolar and Cauchy theories are high, this large stress concentration can be very detrimental for a class of materials such as hollow glass cylinder (sphere) reinforced polymers (Huang and Gibson, 1993), this large stress concentration in the glass shell can trigger early damage of the hollow glass cylinders (spheres). This high stress concentration can be reduced by decreasing the size of hollow glass cylinder.

4. Comparison with average equivalent inclusion method and exact relation

In the following, we consider a fiber-matrix system (without interphase), the fiber is a Cauchy material and the matrix is a micropolar one. For a Cauchy material, Eshelby derived the solution for a homogeneous material in which an ellipsoidal region is subjected to a uniform eigenstrain. Eshelby's result is widely used to determine the stress in an ellipsoidal inhomogeneity, and to compute the effective modulus of composite materials, and this is usually called equivalent inclusion method (Mura, 1982). Recently Cheng and He (1995, 1997) obtained the solution for a homogeneous micropolar material in which a cylindrical or a spherical region is subjected to a uniform eigenstrain and a uniform eigentorsion. According to Cheng and He (1995, 1997), the resulted strain and torsion in a cylindrical region subjected to a uniform eigenstrain $\varepsilon^T_{\alpha\beta}$ and eigentorsion $k^T_{\alpha\beta}$ can be expressed by

$$\varepsilon_{\alpha\beta}(x) = K_{\alpha\beta\lambda\varsigma}(x)\varepsilon^T_{\lambda\varsigma} + L_{\alpha\beta\lambda\varsigma}(x)k^T_{\lambda\varsigma}, \tag{19a}$$

$$k_{\alpha3}(x) = \hat{K}_{\alpha3\lambda3}(x)\varepsilon^T_{\lambda3} + \hat{L}_{\alpha3\lambda3}(x)k^T_{\lambda3}, \tag{19b}$$

where the tensors $K_{\alpha\beta\lambda\varsigma}$, $L_{\alpha\beta\lambda\varsigma}$, $\hat{K}_{\alpha3\lambda3}$ and $\hat{L}_{\alpha3\lambda3}$ are called the micropolar Eshelby tensors, firstly derived by Cheng and He (1997).

The major difference from a Cauchy material is that the resulted stress and couple stress in a circular region are not uniform even for a uniform eigenstrain and a uniform eigentorsion. This makes the equivalent inclusion method widely used for a Cauchy composite difficult to be applied exactly for a micropolar material. For example, a cylindrical inhomogeneity in an infinite micropolar matrix, its effect on stress and couple stress distributions cannot be simulated by the same form inclusion with *uniform* eigenstrain and eigentorsion, rather with some *non-uniform* eigenstrain and eigentorsion.

With this limitation in mind, Sharma and Dasgupta (2002), Liu and Hu (in press) have postulated that the equivalent inclusion method could be applied in an average sense for a micropolar material, and it is called, in the following, an average equivalent inclusion method (AEIM), which is usually utilized for evaluating the average stress for a debonding particle or a reinforced phase of non ellipsoidal form in the case of Cauchy composites. For any remote uniform stress and couple stress $\Sigma_{\alpha\beta}$, $M_{\alpha\beta}$, they are related to a strain and a torsion by $\Sigma_{\alpha\beta} = C^3_{\alpha\beta\lambda\varsigma}E_{\lambda\varsigma}$, $M_{\alpha3} = D^3_{\alpha3\lambda3}K_{\lambda3}$, and $C^3_{\alpha\beta\lambda\varsigma}$, $D^3_{\alpha3\lambda3}$ are the in plane modulus tensors for the matrix material. Following the same idea as in a Cauchy composite, the average stress and couple stress in the fiber can be evaluated from the following equations for a micropolar material, here for completeness, the fiber is also considered to be a micropolar material characterized by its in plane moduli $C^1_{\alpha\beta\lambda\varsigma}, D^1_{\alpha3\lambda3}$ respectively:

$$\langle\sigma_{\alpha\beta}\rangle_1 = \Sigma_{\alpha\beta} + C^3_{\alpha\beta\lambda\varsigma}(\langle\varepsilon_{\lambda\varsigma}\rangle_1 - \varepsilon^T_{\lambda\varsigma}), \tag{20a}$$

$$\langle m_{\alpha3}\rangle_1 = M_{\alpha3} + D^3_{\alpha3\lambda3}(\langle k_{\lambda3}\rangle_1 - k^T_{\lambda3}), \tag{20b}$$

$$\langle \varepsilon_{\alpha\beta}(x)\rangle_1 = E_{\alpha\beta} + \langle K_{\alpha\beta\lambda\varsigma}(x)\rangle_1 \varepsilon^T_{\lambda\varsigma} + \langle L_{\alpha\beta\lambda\varsigma}(x)\rangle_1 k^T_{\lambda\varsigma}, \tag{21a}$$

$$\langle k_{\alpha 3}(x)\rangle_1 = K_{\alpha 3} + \langle \hat{K}_{\alpha 3\lambda 3}(x)\rangle_1 \varepsilon^T_{\lambda 3} + \langle \hat{L}_{\alpha 3\lambda 3}(x)\rangle_1 k^T_{\lambda 3}, \tag{21b}$$

$$\langle \sigma_{\alpha\beta}\rangle_1 = C^1_{\alpha\beta\lambda\varsigma}\langle \varepsilon_{\lambda\varsigma}\rangle_1, \tag{22a}$$

$$\langle m_{\alpha 3}\rangle_1 = D^1_{\alpha 3\lambda 3}\langle k_{\lambda 3}\rangle_1. \tag{22b}$$

$\langle \bullet \rangle_1$ means the average of the said quantity over the fiber region, superscript 1, and 3 refer to the quantity associated with the fiber and the matrix respectively. For a spherical inhomogeneity, and a centro-symmetric and isotropic matrix as shown by Liu and Hu (in press), and for a circular cylinder, we have also shown that the following properties hold

$$\langle L_{\alpha\beta\lambda\varsigma}(x)\rangle_1 = \langle \hat{K}_{\alpha 3\lambda 3}(x)\rangle_1 = 0. \tag{23}$$

These imply, in an average sense, that a uniform eigenstrain produces only an average strain, and a uniform eigentorsion leads only to an average torsion, they are uncoupled for a circular and spherical inhomogeneities.

For a circular fiber, after some lengthy mathematical manipulation, very simple expressions of the average micropolar Eshelby tensors are derived as

$$\langle K_{\alpha\beta\lambda\varsigma}(x)\rangle_1 = T_1\delta_{\alpha\beta}\delta_{\lambda\varsigma} + (T_2+T_3)\delta_{\alpha\lambda}\delta_{\beta\varsigma} + (T_2-T_3)\delta_{\alpha\varsigma}\delta_{\beta\lambda}, \tag{24a}$$

$$\langle \hat{L}_{\alpha 3\lambda 3}(x)\rangle_1 = Q_{33}\delta_{\alpha\lambda}, \tag{24b}$$

where

$$T_1 = \frac{\lambda_3-\mu_3}{4(\lambda_3+2\mu_3)} + \frac{\kappa_3}{2(\kappa_3+\mu_3)} I_1(R_1/g)K_1(R_1/g), \tag{25a}$$

$$T_2 = \frac{\lambda_3+3\mu_3}{4(\lambda_3+2\mu_3)} - \frac{\kappa_3}{2(\kappa_3+\mu_3)} I_1(R_1/g)K_1(R_1/g), \tag{25b}$$

$$T_3 = \frac{4\kappa_3+\mu_3}{2\mu_3} - \frac{2(2\kappa_3+\mu_3)^2}{\mu(\kappa_3+\mu_3)} I_1(R_1/g)K_1(R_1/g), \tag{25c}$$

$$Q_{33} = \frac{\beta_3(\kappa_3+\mu_3)}{4g^2\kappa_3\mu_3} I_1(R_1/g)K_1(R_1/g) \tag{25d}$$

and $g^2 = (\mu_3+\kappa_3)\beta_3/4\mu_3\kappa_3$.

The above formulations are valid for the case where both the matrix and fiber are micropolar materials. Now, we assume the fiber is a Cauchy material, and the matrix is a micropolar one, characterizing respectively by

$$C^1_{\alpha\beta\lambda\varsigma} = \lambda_1\delta_{\alpha\beta}\delta_{\lambda\varsigma} + \mu_1(\delta_{\alpha\lambda}\delta_{\beta\varsigma} + \delta_{\alpha\varsigma}\delta_{\beta\lambda}), \tag{26a}$$

$$C^3_{\alpha\beta\lambda\varsigma} = \lambda_3\delta_{\alpha\beta}\delta_{\lambda\varsigma} + (\mu_3+\kappa_3)\delta_{\alpha\lambda}\delta_{\beta\varsigma} + (\mu_3-\kappa_3)\delta_{\alpha\varsigma}\delta_{\beta\lambda}. \tag{26b}$$

Any isotropic fourth order tensor $H_{\alpha\beta\lambda\varsigma}$ can be always written as a sum of a symmetric and anti-symmetric parts as

$$H_{\alpha\beta\lambda\varsigma} = h_1\delta_{\alpha\beta}\delta_{\lambda\varsigma} + (h_2+h_3)\delta_{\alpha\lambda}\delta_{\beta\varsigma} + (h_2-h_3)\delta_{\alpha\varsigma}\delta_{\beta\lambda} = H^s_{\alpha\beta\lambda\varsigma} + H^a_{\alpha\beta\lambda\varsigma}, \tag{27}$$

where

$$H^s_{\alpha\beta\lambda\varsigma} = \frac{1}{2}(H_{\alpha\beta\lambda\varsigma} + H_{\alpha\beta\lambda\gamma}) = h_1\delta_{\alpha\beta}\delta_{\gamma\varsigma} + h_2(\delta_{\alpha\lambda}\delta_{\beta\varsigma} + \delta_{\alpha\varsigma}\delta_{\beta\lambda}), \tag{28a}$$

$$H^a_{\alpha\beta\lambda\varsigma} = \frac{1}{2}(H_{\alpha\beta\lambda\varsigma} - H_{\alpha\beta\lambda\gamma}) = h_3(\delta_{\alpha\lambda}\delta_{\beta\varsigma} - \delta_{\alpha\varsigma}\delta_{\beta\lambda}). \tag{28b}$$

The advantage of separating a fourth order isotropic tensor into its symmetric and anti-symmetric parts can be explained as follows: since the effect of the eigenstrain and eigentorsion is uncoupled when only a remote uniform symmetric stress (without couple stress) is applied, the average stress in a circular inhomogeneity can be obtained by using the relations for the average stress and strain in Eqs. (20)–(22). Now further split these relations of the average stress and strain into symmetric and anti-symmetric parts, for the symmetric part, we can follow exactly the same method as the classical Eshelby's equivalent inclusion method just replacing the Eshelby tensor by the corresponding micropolar one. If the remote applied stresses are symmetric, the resulted average stresses in a fiber are also symmetric, and the average anti-symmetric stress and the couple stress are zero. So following the classical Eshelby's method (Mura, 1982; Nemat-Nasser and Hori, 1993). For any remote uniform symmetric stress $\Sigma_{(\alpha\beta)}$, subscript ( ) means a symmetrization operator for the indices inside, this stress can be further separated into its deviatoric and spherical parts as $\Sigma_{(\alpha\beta)} = S_{\alpha\beta} + \frac{1}{2}\Sigma_{\gamma\gamma}\delta_{\alpha\beta}$. We also separate the symmetric part $\sigma_{(\alpha\beta)}$ of the local stress $\sigma_{\alpha\beta}$ into its deviatoric and spherical parts $\sigma_{(\alpha\beta)} = s_{\alpha\beta} + \frac{1}{2}\sigma_{\gamma\gamma}\delta_{\alpha\beta}$, so the average stress in the fiber $\langle\sigma_{\alpha\beta}\rangle_1 = \langle\sigma_{(\alpha\beta)}\rangle_1 = \langle s_{\alpha\beta}\rangle_1 + \frac{1}{2}\langle\sigma_{\gamma\gamma}\rangle_1\delta_{\alpha\beta}$ can be related to the remote applied stress $\Sigma_{(\alpha\beta)}$ by

$$\langle s_{\alpha\beta}\rangle_1 = \frac{1}{\mu_3/\mu_1 + (1-\mu_3/\mu_1)\langle K^s_{1212}\rangle_1} S_{\alpha\beta}, \tag{29a}$$

$$\langle\sigma_{\gamma\gamma}\rangle_1 = \frac{1}{k_3/k_1 + (1-k_3/k_1)\langle K^s_{\alpha\alpha\beta\beta}\rangle_1} \Sigma_{\gamma\gamma}. \tag{29b}$$

where $\langle K^S_{1212}\rangle_1$, $\langle K^S_{\alpha\alpha\beta\beta}\rangle_1$ are the components of the symmetric part of average Eshelby tensor, they are

$$\langle K^S_{1212}\rangle_1 = T_2 = \frac{3\mu_3 + \lambda_3}{4(2\mu_3 + \lambda_3)} - \frac{\kappa_3}{2(\mu_3 + \kappa_3)} I_1(R_1/g)K_1(R_1/g), \tag{30a}$$

$$\langle K^S_{\alpha\alpha\beta\beta}\rangle_1 = 4(T_1 + T_2) = \frac{2(\mu_3 + \lambda_3)}{2\mu_3 + \lambda_3}. \tag{30b}$$

It is easy to check that when fiber's diameter tends to infinity, Eqs. (29) and (30) reduce to the corresponding results for a Cauchy material.

The average stress and couple stress in a fiber determined by the exact method (presented in Section 2) will be given in Section 5 for a coated fiber, and the average stress in the fiber for a fiber-matrix system can be obtained as a special case. The main results are that the average couple stress and anti-symmetric stress are zero in a fiber, and only average symmetric stresses are present if only a remote uniform symmetric stress is prescribed. In the following, we will compare the prediction by the approximate equivalent inclusion method (Eq. (29)) with that obtained by the exact method (Sections 2 and 5), only uniaxial loading $\Sigma_{11} \neq 0$ is considered. Fig. 6a and b shows the average stresses in a fiber predicted by the two methods as a function of parameter $\eta = R_1/c_3$ for a hard and soft fiber respectively. In the computation, we take $v_1 = 1/3$, $v_3 = 1/4$, $d_3 = 0.3$. The ratio of the shear modulus of the fiber and the matrix will be specified in the figure.

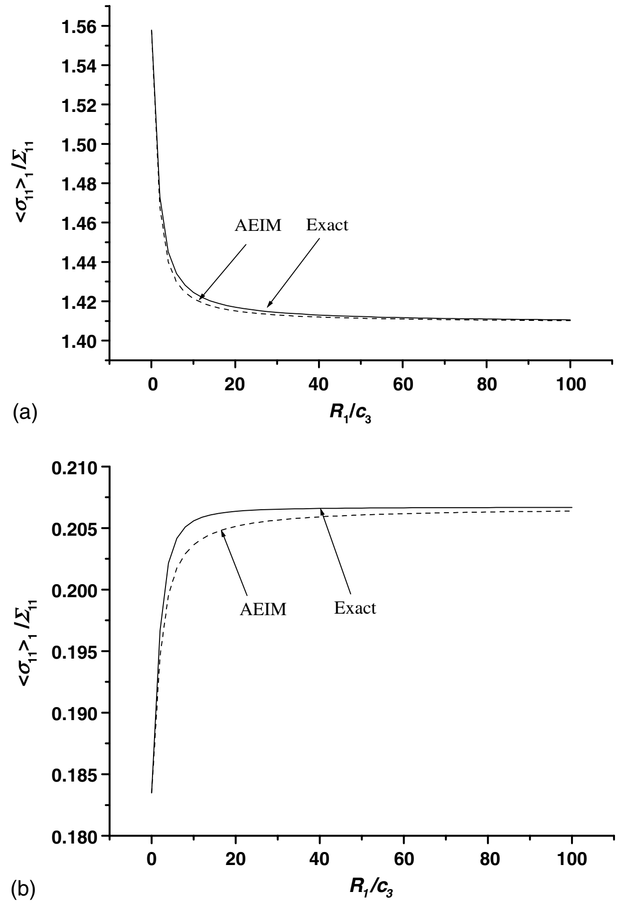

Fig. 6. Comparison of the average stress in a fiber evaluated by exact method and by AEIM method for (a) a hard fiber $\mu_1{:}\mu_3 = 6.4$; (b) a soft fiber $\mu_1{:}\mu_2 = 0.064$.

As shown in Fig. 6, the prediction on the average stress in the fiber based on the approximate average equivalent method agrees well with that obtained from the exact solution, the size dependence of the average fiber stress can be predicted correctly by AEIM method.

5. Effective in plane moduli of a micropolar composite

5.1. Micro–macro transition principle

With the determined local stress presented in Section 2, it is of interest to examine the effective in plane moduli of a micropolar composite. We are only interested in the classical modulus defined between the average symmetric stress and strain, a micro–macro transition principle corresponding to this loading condition will be briefly presented in the following for a two-dimensional case. We are interested in the case where the size of RVE is sufficiently small compared to structural size, so that the macroscopic stress gradient can be neglected on the boundary of RVE. This means only classical loading condition is prescribed on the boundary of RVE (∂RVE) such that

$$u_\alpha = E_{(\alpha\beta)} x_\beta \quad \phi_3 = 0. \tag{31}$$

Here $E_{(\alpha\beta)}$ is a constant and symmetric tensor over RVE, x_β is a coordinate. If one is interested in the full complete moduli of the micropolar composite, a more general loading condition must be examined. For example, in addition to the affine displacement boundary condition, an affine micro-rotation condition must be also prescribed on the boundary. The full effective moduli of a micropolar composite can be obtained by the solution of such boundary value problem and a proper homogenization technique. This at present can only be analyzed by a numerical technique, since the average couple stress is not a simple summation of local couple stress, the contribution of local stresses to the overall moment must be included. The homogenization of this genre is relevant for small scale structures, for example, thin films, where the size of RVE is comparable to the structural size, and this is out of the scope of this paper.

Under the boundary condition (31), for any statically balanced local stresses $(\sigma_{\alpha\beta}, m_{\alpha 3})$ and geometrically compatible local strain fields $(\varepsilon_{\alpha\beta}, k_{\alpha 3})$, the volume average of the strain energy of RVE is

$$\begin{aligned}\langle \sigma_{\alpha\beta}\varepsilon_{\alpha\beta} + m_{\alpha 3}k_{\alpha 3}\rangle &= \langle \sigma_{\alpha\beta}(u_{\beta,\alpha} - e_{3\alpha\beta}\phi_3)\rangle + \langle m_{\alpha 3}\phi_{3,\alpha}\rangle = \frac{1}{V}\int_{\partial \mathrm{RVE}} \sigma_{\alpha\beta}u_\beta n_\alpha \,\mathrm{d}S + \frac{1}{V}\int_{\partial \mathrm{RVE}} m_{\alpha 3}\phi_3 n_\alpha \,\mathrm{d}S \\ &= E_{(\gamma\beta)}\frac{1}{V}\int_{\partial \mathrm{RVE}} \sigma_{\alpha\beta}x_\gamma n_\alpha \,\mathrm{d}S = E_{(\gamma\beta)}\langle \sigma_{\gamma\beta}\rangle = E_{(\alpha\beta)}\langle \sigma_{(\alpha\beta)}\rangle.\end{aligned} \tag{32}$$

On the other hand

$$\langle \varepsilon_{(\alpha\beta)}\rangle = \langle (\varepsilon_{\alpha\beta} + \varepsilon_{\beta\alpha})/2\rangle = \langle (u_{\alpha,\beta} + u_{\beta,\alpha})/2\rangle = E_{(\alpha\beta)} \tag{33}$$

$\langle \bullet \rangle$ means the average of the said quantity over RVE.

If a stress boundary condition is applied on the boundary of RVE such that

$$\sigma_{\alpha\beta}n_\alpha = \Sigma_{(\alpha\beta)}n_\alpha, \quad m_{\alpha 3}n_\alpha = 0, \tag{34}$$

where n_α is the outer normal of the boundary of RVE.

The energy equivalence (Eq. (32)) now can be written as

$$\langle \sigma_{\alpha\beta}\varepsilon_{\alpha\beta} + m_{\alpha 3}k_{\alpha 3}\rangle = \Sigma_{(\alpha\beta)}\langle \varepsilon_{(\alpha\beta)}\rangle \tag{35}$$

and

$$\langle \sigma_{(\alpha\beta)}\rangle = \Sigma_{(\alpha\beta)}. \tag{36}$$

So we can define the classical effective in plane modulus tensor $\overline{C}_{\alpha\beta\gamma\varsigma}$ or compliance tensor $\overline{S}_{\alpha\beta\gamma\varsigma}$ for a micropolar composite by

$$\langle \sigma_{\alpha\beta}\varepsilon_{\alpha\beta} + m_{\alpha 3}k_{\alpha 3}\rangle = E_{(\alpha\beta)}\overline{C}_{\alpha\beta\gamma\varsigma}E_{(\gamma\varsigma)} = \Sigma_{(\alpha\beta)}\overline{S}_{\alpha\beta\gamma\varsigma}\Sigma_{(\gamma\varsigma)}. \tag{37}$$

In order to determine the effective modulus tensor $\overline{C}_{\alpha\beta\gamma\varsigma}$, we will apply a displacement boundary condition (31) and determine the average symmetric stress, and then the relation between average symmetric stress $\langle \sigma_{(\alpha\beta)}\rangle$ and the average symmetric strain $E_{(\alpha\beta)}$ provides the classical effective in plane moduli of a micropolar composite.

5.2. Effective in plane moduli of a micropolar fiber composite

5.2.1. Effective moduli determined by AEIM method for a two-phase fiber composite

Effective in plane moduli for a composite with classical fibers (without interphase) and a micropolar matrix will be estimated by the approximate equivalent inclusion method. As shown in Section 4, the average stress and couple stress in a fiber are uncoupled for a remote uniform symmetric stress, we can follow exactly the same method as for a Cauchy composite, except that the Eshelby tensor must be replaced by the new one given by Eq. (30). In this paper, the Mori–Tanaka's method (see for example, Hu and Weng, 2000a,b) will be utilized to derive the relation between the average symmetric stress and strain of RVE, finally the effective shear and in plane bulk moduli of a micropolar fiber composite are respectively

$$\mu_c = \mu_3\left\{1 + \frac{f_1}{2(1-f)\langle K^S_{1212}\rangle_1 + [\mu_3/(\mu_1-\mu_3)]}\right\}, \tag{38a}$$

$$k_c = k_3\left\{1 + \frac{f_1}{[(1-f)\langle K^S_{\alpha\alpha\beta\beta}\rangle_1/2] + [k_3/(k_1-k_3)]}\right\}. \tag{38b}$$

where f_1 is the volume fraction of fibers.

Since $\langle K^S_{\alpha\alpha\beta\beta}\rangle_1 = 2(\mu_3+\lambda_3)/(2\mu_3+\lambda_3)$, which is identical to the spherical part of the classical Eshelby tensor for a circular fiber in a plane strain condition, so the predicted effective in plane bulk modulus is the same as that for the corresponding Cauchy composite. However the effective shear modulus depends on fiber's size through the component of the average micropolar Eshelby tensor $\langle K^S_{1212}\rangle_1$, which is given by Eq. (30a), with a classical term (Cauchy medium) and a term related to the micropolar effect of the matrix.

5.2.2. Effective in plane moduli determined with the exact localization relation

As shown in Section 4 for the average equivalent inclusion method, a single circular fiber embedded in an infinite micropolar matrix under a remote constant stress and couple stress, the average stress in the fiber is only related to the applied remote stress, and the remote couple stress produces only an average couple stress in the fiber. Encouraged by the results obtained by AEIM method, here we assume the remote couple stress produces only an average couple stress in the fiber. The localization relation of Mori–Tanaka's method is obtained by embedding a fiber (or coated fiber) into the matrix material under yet unknown average stress and couple stress of the matrix. With the previous assumption (effect of average stress and couple stress is uncoupled in the fiber), only the relations of the average stress and strain is considered if we are concerned with the classical effective in plane modulus of the composite. In the following, the average stresses in the fiber and interphase will be evaluated for a remote constant and symmetric stress.

(i) hydrostatic loading

Now let the remote stresses be $\Sigma_{11} = \Sigma_{22} = \Sigma$, it is shown from the results presented in Section 2 that the average stresses in the fiber and in the interphase can be expressed as

$$\begin{aligned} &\langle\sigma_{11}\rangle_i = 2A^i_2 = s_i\Sigma, \\ &\langle\sigma_{22}\rangle_i = \langle\sigma_{11}\rangle_i, \quad \langle\sigma_{12}\rangle_i = \langle\sigma_{21}\rangle_i = 0, \quad \langle m_{13}\rangle_2 = \langle m_{23}\rangle_2 = 0, \end{aligned} \tag{39}$$

where i ranges from 1 to 2, referring respectively to the fiber and the interphase. It is found that the average couple stresses in the fiber and the interphase regions are zero.

(ii) shear loading

The following remote loading condition $\Sigma_{11} = -\Sigma_{22}$ is applied, with the help of the solution given in Sections 2 and 3, and averaging the stress and couple stress over the fiber and the interphase regions, after a lengthy mathematical manipulation, we get finally the average stress and couple stress in the classical fiber

$$\begin{aligned} &\langle\sigma_{11}\rangle_1 = -(2A^1_4 + 2A^1_8 + 3A^1_6) - I_1(R_1/c_1)R_1A^1_{10}/2c_1 = p_1\Sigma_{11}, \\ &\langle\sigma_{22}\rangle_1 = \langle\sigma_{22}\rangle_1, \quad \langle\sigma_{12}\rangle_1 = \langle\sigma_{21}\rangle_1 = 0. \end{aligned} \tag{40}$$

The average stress and couple stress of the interphase are

$$\begin{aligned}\langle\sigma_{11}\rangle_2 &= -[2A_4^2+3(1+t^2)A_6^2+2A_8^2]-\frac{R_1}{2c_2(t^2-1)}\{[-K_1(R_1/c_2)+tK_1(R_1t/c_2)]A_9^2\\ &\quad+[I_1(R_1/c_2)-tI_1(R_1t/c_2)]A_{10}^2\}\\ &= p_2\Sigma_{11},\end{aligned}$$

$$\langle\sigma_{22}\rangle_2=\langle\sigma_{11}\rangle_2,\langle\sigma_{12}\rangle_2=\langle\sigma_{21}\rangle_2=0,\langle m_{13}\rangle_2=\langle m_{23}\rangle_2=0, \tag{41}$$

where $t = R_2/R_1$.

So under a remote constant pure shear stress, the average stress in the central fiber and the external layer are also of pure shear. For a finite concentration of coated fibers, Mori–Tanaka's mean field theory is utilized to consider their interactions. Now embed one single coated fiber into a micropolar matrix under a remote unknown matrix stress $\langle\sigma_{(\alpha\beta)}\rangle_3$ (we do not consider the average couple stress of the matrix, since it contributes nothing on average to the stress in the fiber and the interphase), $\langle\bullet\rangle_3$ means the volume average of the said quantity over the matrix, the average stresses in a fiber and in an interphase can be evaluated with the method described previously.

With the help of the following homogenization relations

$$\Sigma_{(\alpha\beta)}=\langle\sigma_{(\alpha\beta)}\rangle=(1-f_1-f_2)\langle\sigma_{(\alpha\beta)}\rangle_3+f_1\langle\sigma_{(\alpha\beta)}\rangle_1+f_2\langle\sigma_{(\alpha\beta)}\rangle_2, \tag{42a}$$

$$\langle\varepsilon_{(\alpha\beta)}\rangle=(1-f_1-f_2)\langle\varepsilon_{(\alpha\beta)}\rangle_3+f_1\langle\varepsilon_{(\alpha\beta)}\rangle_1+f_2\langle\varepsilon_{(\alpha\beta)}\rangle_2 \tag{42b}$$

and $\langle\sigma_{(\alpha\beta)}\rangle_3=C^{3s}_{\alpha\beta\lambda\varsigma}\langle\varepsilon_{(\lambda\varsigma)}\rangle_3$, where $C^{3s}_{\alpha\beta\lambda\varsigma}$ is the symmetric part of the modulus tensor $C^{3}_{\alpha\beta\lambda\varsigma}$of the matrix material. We can eliminate the unknown average matrix stress $\langle\sigma_{(\alpha\beta)}\rangle_3$, then the relation between $\Sigma_{(\alpha\beta)}$ and $\langle\varepsilon_{(\alpha\beta)}\rangle$ gives the classical symmetric in plane modulus tensor for a micropolar composite, the final results are for the effective shear and in plane bulk moduli

$$\mu_c=\frac{f_1p_1+f_2p_2+(1-f_1-f_2)}{f_1\frac{p_1}{\mu_1}+f_2\frac{p_2}{\mu_2}+(1-f_1-f_2)\frac{1}{\mu_3}}, \tag{43a}$$

$$k_c=\frac{f_1s_1+f_2s_2+(1-f_1-f_2)}{f_1\frac{s_1}{k_1}+f_2\frac{s_2}{k_2}+(1-f_1-f_2)\frac{1}{k_3}}, \tag{43b}$$

where f_1, f_2 are the volume fractions of fibers and interphase respectively. The relations (43) have the same form as the effective shear and in plane bulk moduli for a composite with a classical matrix and coated fibers estimated by Mori–Tanaka's method (Hu, 1997), however the stress concentration coefficients $p_1, p_2; s_1, s_2$ must be determined by the corresponding material model.

Now we will firstly examine a two-phase composite (without interphase), the material constants used in the computation are: $\mu_1{:}\mu_3 = 6.4$, $v_1 = 1/3$, $v_3 = 1/4$ and $d_3 = 0.3$.

The influence of fiber's diameter on the effective shear modulus for a two-phase fiber composite (without interphase) is presented in Fig. 7 as a function of parameter $\eta = R_1/c_3$ for different volume fractions of fibers. The predictions based on AEIM method and on Cauchy theory are also included for the comparison.

Compared to the classical theory in which no scale effect is present, the effective shear modulus predicted by micropolar theory depends on fiber's diameter, it coincides with that predicted by Cauchy theory for large η (large fiber's diameter), however when the fiber's diameter is comparable to the matrix characteristic length (characterized by c_3), the prediction based on micropolar theory deviates rapidly from that predicted by Cauchy theory. The predictions on the effective shear modulus by AEIM method and by the exact localization relation agree well with each other.

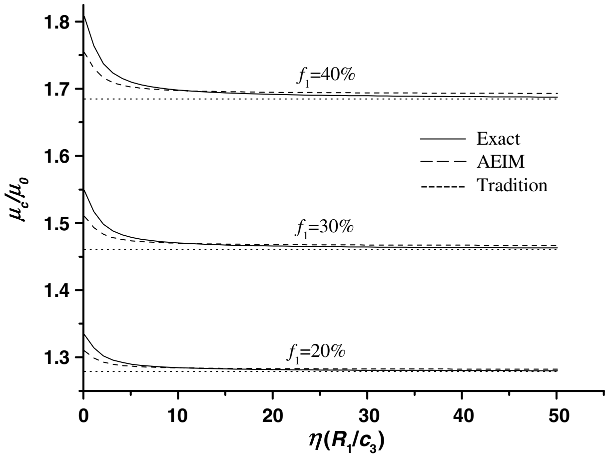

Fig. 7. Effective in plane shear modulus as a function of parameter $\eta = R_1/c_3$ for different volume fractions of fiber.

In the presence of an interphase layer, only the comparison with the classical prediction is shown in Fig. 8 as a function of parameter $\eta = R_1/c_2$ for different volume fractions of fibers. The material constants used for the computation are: μ_1:μ_2:μ_3 = 84:13:1, $v_1 = 0.25$, $v_2 = 0.35$, $v_3 = 0.4$, $R_2 = 1.1R_1$, $d_2 = 0.3$, $d_3 = 0.2$ and further we assume $c_2 = c_3$.

As shown in Fig. 8, the size dependence of the effective shear modulus of a composite with coated fibers is the same as that for a two-phase fiber composite (without interphase), micropolar theory predicts a sharp increase of the effective shear modulus when the fiber's diameter is comparable to the material's characteristic length ($c_2 = c_3$), and the classical theory is scale independent. Finally for the effective in plane bulk modulus, we find that the prediction based on micropolar theory is identical to that predicted by Cauchy theory for both fiber-matrix system and fiber, interphase and matrix system. This can be expected, since in micropolar theory, only rigid rotation is considered for the microstructure, the dilatation effect of the microstructure is not included. Finally it should be mentioned that the determination of the material

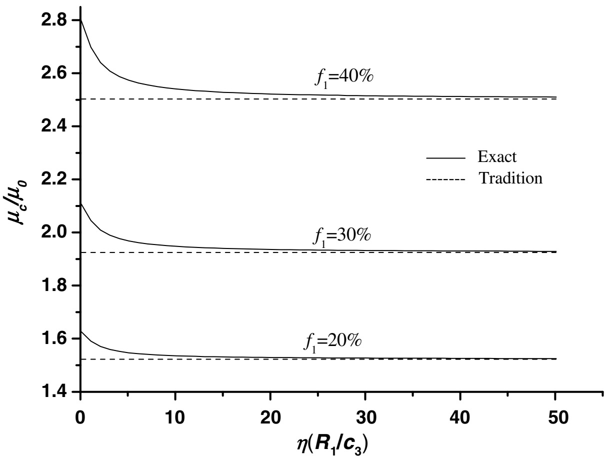

Fig. 8. Effective shear modulus of the composite with coated fibers as a function of parameter $\eta = R_1/c_2$ for different fiber's volume fractions.

constants for a micropolar material still remains as a challenge, the different techniques and methods can be found in the review given by Lake (1995).

6. Conclusions

We have therefore proposed an analytical method to evaluate the local stress and couple stress for a coated fiber in a micropolar matrix under a remote constant and symmetric stress. Compared to the classical prediction for a Cauchy material, the micropolar effect of the interphase has an important influence on the stress distributions in the interphase region and in the fiber. The determined average stress in the fiber for a two-phase composite (without interphase) is also compared to that obtained by the approximate average equivalent inclusion method, a good agreement is found for these two methods, so the approximate AEIM method can be used to evaluate the average stress in a fiber and to construct the effective moduli of a micropolar composite. A micro–macro transition method is also proposed, the classical effective in plane moduli of a micropolar composite, relating the average symmetric stress and strain of RVE, are determined analytically. It is found that the effective in plane bulk modulus is identical to that determined by Cauchy theory, however the effective shear modulus depends on the fiber's diameter, for large fiber's diameter the effective shear modulus predicted by micropolar theory coincides with that predicted by Cauchy theory, as required.

Acknowledgement

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Appendix A

The expression for the matrix $M_i(r)$

$$\begin{bmatrix}
\frac{R_1^2}{r^2}, 2, 0, 0, 0, 0, 0, 0, 0, 0 \\
\frac{-R_1^2}{2\mu_2 r}, \frac{r}{\mu_2+\lambda_2}, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, -\frac{4R_1^2}{r^2}, -2, -\frac{6R_1^4}{r^4}, 0, \frac{6R_1^4}{r^4}, -2, \frac{6R_1^{2^2}}{r^2} K_0\left(\frac{r}{c_i}\right) + \left(\frac{12c_iR_1^2}{r^3} + \frac{2R_1^2}{c_i r}\right) K_1\left(\frac{r}{c_i}\right), \frac{6R_1^2}{r^2} I_0\left(\frac{r}{c_i}\right) - \left(\frac{12c_iR_1^2}{r^3} + \frac{2R_1^2}{c_i r}\right) I_1\left(\frac{r}{c_i}\right) \\
0, 0, -\frac{2R_1^2}{r^2}, 2, -\frac{6R_1^4}{r^4}, \frac{6r^2}{R^2}, \frac{6R_1^4}{r^4}, 2, \frac{6R_1^2}{r^2} K_0\left(\frac{r}{c_i}\right) + \left(\frac{12c_iR_1^2}{r^3} + \frac{2R_1^2}{c_i r}\right) K_1\left(\frac{r}{c_i}\right), \frac{6R_1^2}{r^2} I_0\left(\frac{r}{c_i}\right) - \left(\frac{12c_iR_1^2}{r^3} + \frac{2R_1^2}{c_i r}\right) I_1\left(\frac{r}{c_i}\right) \\
0, 0, 0, 0, 0, 0, -\frac{2R_1^4}{r^3}, 2r, -\frac{2R_1^2}{r} K_0\left(\frac{r}{c_i}\right) - \left(\frac{4c_iR_1^2}{r^2} + \frac{R_1^2}{c_i}\right) K_1\left(\frac{r}{c_i}\right), -\frac{2R_1^2}{r} I_0\left(\frac{r}{c_i}\right) + \left(\frac{4c_iR_1^2}{r^2} + \frac{R_1^2}{c_i}\right) I_1\left(\frac{r}{c_i}\right) \\
0, 0, \frac{R_1^2(\lambda_i+2\mu_i)}{r\mu_i(\lambda_i+\mu_i)}, -\frac{r}{\mu_i}, \frac{R_1^4}{r^3\mu_i}, -\frac{r^3\lambda_i}{R_1^2\mu_i(\lambda_i+\mu_i)}, -\frac{R_1^4}{r^3\mu_i}, -\frac{r}{\mu_i}, -\frac{R_1^2}{r\mu_i} K_0\left(\frac{r}{c_i}\right) - \frac{2c_iR_1^2}{r^2\mu_i} K_1\left(\frac{r}{c_i}\right), -\frac{R_1^2}{r\mu_i} I_0\left(\frac{r}{c_i}\right) + \frac{2c_iR_1^2}{r^2\mu_i} I_1\left(\frac{r}{c_i}\right) \\
0, 0, \frac{-R_1^2}{r\mu_i(\lambda_i+\mu_i)}, \frac{r}{\mu_i}, \frac{R_1^4}{r^3\mu_i}, \frac{r^3(2\lambda_i+3\mu_i)}{R_1^2\mu_i(\lambda_i+\mu_i)}, -\frac{R_1^4}{r^3\mu_i}, \frac{r}{\mu_i}, -\frac{R_1^2}{r\mu_i} K_0\left(\frac{r}{c_i}\right) - \frac{(4c_i^2+r^2)R_1^2}{2r^2c_i\mu_i} K_1\left(\frac{r}{c_i}\right), -\frac{R_1^2}{r\mu_i} I_0\left(\frac{r}{c_i}\right) + \frac{(4c_i^2+r^2)R_1^2}{2r^2c_i\mu_i} I_1\left(\frac{r}{c_i}\right) \\
0, 0, 0, 0, 0, 0, \frac{R_1^4}{r^2\beta_i}, \frac{r^4}{\beta_i}, \frac{R_1^2}{\beta_i} K_0\left(\frac{r}{c_i}\right) + \frac{2c_iR_1^2}{r\beta_i} K_1\left(\frac{r}{c_i}\right), \frac{R_1^2}{\beta_i} I_0\left(\frac{r}{c_i}\right) - \frac{2c_iR_1^2}{r\beta_i} I_1\left(\frac{r}{c_i}\right)
\end{bmatrix}$$

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