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Identification of material parameters of micropolar theory for composites by homogenization method

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ABSTRACT

The objective of the present work is to develop an analytical homogenization method to derive higher-order material parameters of micropolar theory. With help of Airy's stress function, the constitutive equations for a homogenized micropolar medium are analytically established by considering a cylindrical representative volume element (RVE) subjected to quadratic and cubic boundary displacement conditions. Both porous and composite materials are considered, an analytical relation between the intrinsic length and the microstructural parameters is given.

1. Introduction

Micropolar theory introduces three additional degrees of freedom for microstructure rotation while still keeping in a continuum formulation, and it can characterize the size effect of materials observed in experiments [1–4]. However, evaluation of the higher-order modulus introduced in the theory is difficult either by experiments or by known homogenization techniques. This dramatically limits applications of micropolar theory although it was initially proposed by Cosserat brothers a century ago [5]. Basically, the micropolar theory is proposed for better characterizing deformations of microstructural materials, so the higher-order modulus can in principle be determined from the local information of the material. For foams and lattice materials, efforts have recently been made to homogenize these materials using the micropolar model, the material constants are derived directly for the microstructure [3,4,6]. For composite materials, Bigoni and Drugan [7] derived the homogenized micropolar constants by applying the quadratic form of the loading. Zybell et al. [8] derived the homogenized strain gradient model for a porous material. In this paper we will follow the idea proposed by Forest [9–11] and choose a cylindrical representative volume element (RVE) under plane strain condition, by applying quadratic and cubic forms of loading on the boundary of the RVE. We will show that the two higher-order moduli can be related analytically to the microstructure and the elastic properties of the composite. The present paper is arranged as follows: The boundary conditions for homogenization within the frame of micropolar theory are discussed in Section 2. Section 3 investigates a cylindrical RVE under quadratic and cubic forms of boundary conditions. Analytical solutions obtained from the RVE are used to determine the higher-order moduli of the composite. The final section includes short conclusions.

2. Boundary conditions for the higher-order homogenization scheme

Within the framework of classical homogenization theory, a RVE is assumed to be large enough to contain typical microstructure of the material, but it must be sufficiently small so that the homogenous stress and strain can be applied on its boundary. However, when these conditions fail to be applied, e.g. the size of RVE is comparable to the microstructure or strong deformation gradients are present, nonlocal homogenization must be adopted.

The linear boundary condition in classical homogenization is assumed as $u_i = E_{(ij)}x_j$ or $\sigma_{ij}n_j = \sum_{(ij)}n_j$, where u_i and σ_{ij} are local displacements and stress components in the RVE respectively, $E_{(ij)}$ and $\sum_{(ij)}$ are symmetric macroscopic strain and stress applied on the boundary of the RVE. The aim of the classical homogenization is

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to replace the heterogeneous materials by a homogeneous Cauchy medium with the Hill–Mandel condition:

$$\langle\sigma_{ij}\varepsilon_{ij}\rangle = \Sigma_{(ij)}E_{(ij)}. \tag{1}$$

where $\langle * \rangle$ means the volume average of $*$ over the RVE. For the non-local homogenization, a question arises: what kind of boundary conditions should be assumed for the RVE?

In this paper a micropolar homogenization scheme proposed by Forest [10,11] is employed, which is based on a systematical construction of relations between the macroscopic higher-order kinematic quantities and local displacements. The governing equations for centro-symmetric micropolar continuum read [1,12]:Equilibrium equations:

$$\Sigma_{ij,i} = 0, \tag{2a}$$
$$M_{ij,i} + e_{ijk}\Sigma_{jk} = 0. \tag{2b}$$

Geometric equations:

$$E_{(ij)} = \frac{1}{2}(U_{j,i} + U_{i,j}), \tag{3a}$$
$$E_{\langle ij\rangle} = \frac{1}{2}(U_{j,i} - U_{i,j}) - e_{ijk}\Phi_k, \tag{3b}$$
$$K_{ij} = \Phi_{j,i}. \tag{3c}$$

Constitutive equations:

$$\Sigma_{ij} = D_{ijkl}E_{kl}, \tag{4a}$$
$$M_{ij} = L_{ijkl}K_{kl}, \tag{4b}$$

where subscripts () and $\langle\ \rangle$ denote the symmetric and antisymmetric part of tensors respectively, U_i and Φ_i are the displacements and micro-rotations, Σ_{ij} and M_{ij} are the non-symmetric stress and couple stress, E_{ij} and K_{ij} the strain and curvature, D_{ijkl} and L_{ijkl} the stiffness tensors.

In order to replace a heterogeneous material by an equivalent micropolar medium, the relation between the local displacement field in the RVE $u_i(x_j)$ and the macroscopic displacements $U_i(X_j)$ and micro-rotations $\Phi_i(X_j)$ can be constructed using the least square approximation. The displacement of the homogenized micropolar element is

$$u_i^*(x_j) = U_i(X_j) + e_{imn}\Phi_m(X_j)(x_n - X_n), \tag{5}$$

where $X_j(j = 1,2,3)$ are the coordinates of the mass center. The best fit of u_i^* to the RVE's real displacement field u_i can be achieved by minimization of the error over the volume V:

$$\Delta = \int_V |u_i(x_j) - u_i^*(x_j)|^2 dV. \tag{6}$$

The minimization is taken over a cubic RVE with length L, which gives

$$U_i = \langle u_i\rangle, \tag{7a}$$
$$\Phi_i = \frac{6}{L^2}\langle e_{imn}(x_m - X_m)u_n\rangle, \tag{7b}$$

then the macroscopic strain and torsion are derived according to the geometric Eq. (3).

The local displacement in the RVE can be expressed in polynomial form as following

$$\begin{aligned} u_i = {} & A_i + B_{i1}x_1 + B_{i2}x_2 + B_{i3}x_3 + C_{i1}x_1^2 + C_{i2}x_2^2 \\ & + C_{i3}x_3^2 + 2C_{i4}x_1x_2 + 2C_{i5}x_2x_3 + 2C_{i6}x_1x_3 + \cdots \end{aligned} \tag{8}$$

The coefficients of the polynomial can be identified in the micropolar framework through Eqs. (3), (7) and (8) providing scale invariance condition $E_{ij}(X_k) = \varepsilon_{ij}(X_k)$ [11]. The following three kinds of the boundary conditions are of special interests:

- If the displacement on the boundary of RVE is linear, this corresponds to classical homogenous boundary condition

$$u_i = E_{(ij)}x_j. \tag{9}$$

- If the displacement is a second-order polynomial, one can show that

$$u_1 = -\frac{1}{2}K_{23}x_2^2 + \frac{1}{2}K_{32}x_3^2 - K_{13}x_1x_2 + K_{12}x_1x_3 + \frac{2}{3}(K_{22} - K_{33})x_2x_3, \tag{10a}$$
$$u_2 = \frac{1}{2}K_{13}x_1^2 - \frac{1}{2}K_{31}x_1^2 + K_{23}x_1x_2 - K_{21}x_1x_3 + \frac{2}{3}(K_{33} - K_{11})x_1x_3, \tag{10b}$$
$$u_3 = -\frac{1}{2}K_{12}x_1^2 + \frac{1}{2}K_{21}x_2^2 + K_{31}x_2x_3 - K_{32}x_1x_3 + \frac{2}{3}(K_{11} - K_{22})x_1x_2. \tag{10c}$$

- If the displacement is a third-order polynomial, it can be expressed as

$$u_1 = 10(\theta_3(x_2^3 - 3x_1^2x_2) + \theta_2(3x_1^2x_3 - x_3^3)), \tag{11a}$$
$$u_2 = 10(\theta_3(-x_1^3 + 3x_2^2x_1) + \theta_1(-3x_2^2x_3 + x_3^3)), \tag{11b}$$
$$u_3 = 10(\theta_2(x_1^3 - 3x_3^2x_1) + \theta_1(3x_3^2x_2 - x_2^3)). \tag{11c}$$

with $\theta_i = \frac{1}{2L^2}e_{ijk}E_{\langle ik\rangle}$.

The results above show that the quadratic terms in the boundary displacement are related to the effect of torsion and the cubic terms are necessary in considering the effect of antisymmetric strain. In order to obtain an equivalent micropolar continuum, terms of up to third-order polynomial need be adopted.

Finally the overall micropolar material properties can be defined by an extended form of Hill–Mandel condition

$$\langle\sigma_{ij}\varepsilon_{ij}\rangle = \frac{1}{V}\int_V \sigma_{ij}u_{j,i}dV = \Sigma_{(ij)}E_{(ij)} + M_{ij}K_{ij} + \Sigma_{\langle ij\rangle}E_{\langle ij\rangle}. \tag{12}$$

By submitting the previous three sets of local displacements u_i (9–11) into (12) it is straight forward to obtain the macroscopic stress and couple stress in terms of local stress field. For example, one component of macroscopic couple stress and antisymmetric stress are given below

$$M_{13} = \frac{1}{V}\int_V (-y\sigma_{11})dV. \tag{13}$$

$$\Sigma_{\langle 12\rangle} = \frac{30}{L^2}\frac{1}{V}\int_V (xy(\sigma_{22} - \sigma_{11}) + (y^2 - x^2)\sigma_{12})dV. \tag{14}$$

Once the local stress field can be obtained from given boundary displacements, the overall micropolar stiffness can then be determined in conjunction with the constitutive Eq. (4).

3. Analysis based on a cylindrical RVE

In this section, a plane strain problem corresponding to an infinitely long cylindrical RVE will be considered. The 2D configuration considered here consists of a circular linear elastic inclusion with radius a surrounded by a matrix material. Given the inclusion's volume fraction f, the radius of RVE $R = a/\sqrt{f}$, we expect that the solution derived is valid for composites with small fraction of inclusion [13].

For this case, the procedure mentioned in Section 2 must be reformed to accommodate the 2D circular RVE region, i.e., $U_\alpha = \langle u_\alpha\rangle$ and $\Phi_3 = \frac{2}{R^2}\langle e_{3\alpha\beta}(x_\alpha - X_\alpha)u_\beta\rangle$ (α, β ranges from 1 to 2) should be used instead of Eq. (7). A straight forward deduction gives the boundary displacements used in the following (Eqs. (18) and (32)).

3.1. A general solution in Fourier series

Considering a plane strain problem, the Airy's stress function is used to derive the general solution in the polar coordinates. The stress function must satisfy the biharmonic equation:

$$\nabla^2\nabla^2 F^i = 0, \tag{15}$$

where ∇^2 denotes Laplace operator. The stresses can be obtained through the Airy's stress function by

$$\sigma^i_{rr} = \frac{1}{r}\frac{\partial F^i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F^i}{\partial \theta^2}, \quad \sigma^i_{\theta\theta} = \frac{\partial^2 F^i}{\partial r^2}, \tag{16a}$$

$$\sigma^i_{r\theta} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F^i}{\partial \theta}\right), \tag{16b}$$

where the superscript i varies from 1 to 2, representing the inclusion and matrix phases, respectively.

The general solution of F^i in polar coordinates is derived by Mitchell [14], corresponding to all kinds of boundary conditions. In present work, up to third-order terms in the boundary polynomials are retained. Due to the linear nature of the problem, the stress function can be split into three parts

$$F^i = F^i_1 + F^i_2 + F^i_3. \tag{17}$$

where F^i_1 corresponds to the linear boundary displacement, F^i_2 and F^i_3 correspond to quadratic and cubic boundary respectively. In classical micromechanics scheme, only F^i_1 is adopted to derive the classical effective properties [15,16]. The higher-order continuum model needs the stress and strain fields related to F^i_2 and F^i_3.

3.2. Quadratic displacement boundary condition

Following the approach explained previously, the quadratic displacement boundary conditions for the cylindrical RVE are applied

$$u_1|_{\partial V} = -K_{13}xy - \frac{1}{2}K_{23}y^2, \tag{18a}$$

$$u_2|_{\partial V} = \frac{1}{2}K_{13}x^2 + K_{23}xy, \tag{18b}$$

where K_{13} and K_{23} are the torsion components for the plane strain problem. The stress function F^i_2 corresponding to the quadratic displacement can be obtained as

$$\begin{aligned} F^i_2 = {} & (A^i_1 r^3 + A^i_2\frac{1}{r} + A^i_3 r\log r)\cos(\theta) \\ & + (A^i_4 r^3 + A^i_5\frac{1}{r} + A^i_6 r\log r)\sin(\theta) \\ & + (A^i_7 r^3 + A^i_8 r^5 + A^i_9 r^{-3} + A^i_{10} r^{-1})\cos(3\theta) \\ & + (A^i_{11} r^3 + A^i_{12} r^5 + A^i_{13} r^{-3} + A^i_{14} r^{-1})\sin(3\theta). \end{aligned} \tag{19}$$

The interface between the matrix and inclusion is assumed to be perfectly bonded, so the continuity and equilibrium conditions across the interface are:

$$u^1_r(a,\theta) = u^2_r(a,\theta), \quad u^1_\theta(a,\theta) = u^2_\theta(a,\theta), \tag{20a}$$

$$\sigma^1_{rr}(a,\theta) = \sigma^2_{rr}(a,\theta), \quad \sigma^1_{r\theta}(a,\theta) = \sigma^2_{r\theta}(a,\theta), \tag{20b}$$

The unknown coefficients $A^i_j (i = 1,2, j = 1,\ldots,14)$ are determined from the interface conditions (20) and the boundary conditions (18) for the RVE. Only four independent non-zero coefficients are retained [8]. Therefore, the stress and strain as well as the boundary displacements and surface traction can be written as [8]

$$\tilde\sigma^s_{ij} = \sum_{k=1}^{4} A_k\tilde\sigma^{sk}_{ij}, \quad \tilde\varepsilon^s_{ij} = \sum_{k=1}^{4} A_k\tilde\varepsilon^{sk}_{ij}, \tag{21a}$$

$$\tilde u^s_i = \sum_{k=1}^{4} A_k\tilde u^{sk}_i, \quad \tilde t^s_i = \sum_{k=1}^{4} A_k\tilde t^{sk}_i. \tag{21b}$$

where $s = 1,2$ denotes the inclusion and matrix, respectively, the unknown coefficients $A_k (k = 1,\ldots,4)$ are the four independent non-zero coefficients. $\tilde\sigma^{sk}_{ij}$, $\tilde\varepsilon^{sk}_{ij}$, $\tilde u^{sk}_i$ and $\tilde t^{sk}_i$ are functions of r and θ. It follows from the orthogonality properties of the trigonometric function that

$$\frac{1}{V}\int_V \tilde\sigma^{sk}_{ij}\tilde\varepsilon^{sk}_{ij}dV = \begin{cases} \tau_k, & k = l, \\ 0, & k \neq l. \end{cases} \tag{22}$$

According to the principle of conservation of energy [8], we have

$$\frac{1}{2V}\int_{\partial V} \tilde u_i \tilde t_i dS = \frac{1}{2V}\int_V \tilde\sigma^s_{ij}\tilde\varepsilon^s_{ij}dV = \frac{1}{2}\sum_{k=1}^{4} A^2_k\tau_k. \tag{23}$$

Submitting the boundary conditions (18) into Eq. (23), the non-zero coefficients A_k are determined as following

$$A_k = \frac{1}{\tau_k}K_{ij}m^k_{ij}, \quad (k = 1,\ldots,4). \tag{24}$$

where the non-zero m^k_{ij} are

$$m^k_{13} = \frac{1}{V}\int_V -y\tilde\sigma^{sk}_{11}dV, \tag{25a}$$

$$m^k_{23} = \frac{1}{V}\int_V x\tilde\sigma^{sk}_{22}dV. \tag{25b}$$

With help of the extended Hill–Mandel condition (12), only considering quadratic boundary conditions, we have

$$\frac{1}{2V}\int_V \tilde\sigma^s_{ij}\tilde\varepsilon^s_{ij}dV = \frac{1}{2}\sum_{k=1}^{4} A^2_k\tau_k = \frac{1}{2}K_{ij}M_{ij}. \tag{26}$$

Therefore, the macroscopic couple stress can be defined by

$$M_{ij} = \sum_{r=1}^{4}\frac{1}{\tau^r}m^r_{ij}m^r_{kl}K_{kl} = L_{ijkl}K_{kl}. \tag{27}$$

Finally, the higher-order modulus can be evaluated by

$$L_{ijkl} = \sum_{r=1}^{4}\frac{1}{\tau^r}m^r_{ij}m^r_{kl}. \tag{28}$$

For the plane strain condition, only non-zero components L_{1313} and L_{2323} are retained, and they are identical. Defining $\beta = L_{1313} = L_{2323}$ as bending moduli, we finally obtain

$$\beta = R^2E(\phi_1(v) + \phi_2(v,\eta_1,\eta_2)f^2 + O(f^3)), \tag{29}$$

where $\phi_1\ (v)$, $\phi_2\ (v,\eta_1,\eta_2)$ are non-dimensional functions of material contrasts $\eta_1 = v_1/v$, $\eta_1 = E_1/E$ and v. Here v and E are Poisson's ratio and Young's modulus of the matrix, respectively, v_1 and E_1 are those for the inclusion.

In higher-order theory of continua, one or more characteristic lengths have been introduced. A sophisticated question is how to interpret their physical meaning and how to relate them to the material microstructure lengths. The solutions we obtained provide an analytical relation between the length parameters of micropolar elasticity and the microstructure of the composite. The characteristic length in the micropolar elasticity is given by

$$l^2_m = \frac{\beta}{E} = R^2(\phi_1(v) + \phi_2(v,\eta_1,\eta_2)f^2 + O(f^3)). \tag{30}$$

In the expression above all variables in the right hand side are known by means of microstructural information of composites. The characteristic length is a quadratic function of the RVE size and a nonlinear function of the volume fraction.

For porous materials ($E_1 = 0$), the above result can be simplified as

$$\beta^p = \frac{R^2E(3+2v)}{2(3+2v-v^2)} - \frac{3R^2E(69+72v+29v^2+2v^3)}{25(-3-3v+v^2)}f^2 + O(f^3). \tag{31}$$

Fig. 1 shows that for a composite material, the normalized β/β^0 becomes larger with increasing f or material contrast η_1, η_2, where $\beta^0 = R^2E(3+2v)/(2(3+2v-v^2))$ is the corresponding quantity for the matrix material ($f = 0$). For porous materials, the normalized β^p/β^0 decreases with the increase of f. When f trends to zero, both β/β^0 and β^p/β^0 become unity as expected.

It must be mentioned here that when f goes to zero, the higher-order moduli β^0 corresponds to the bending stiffness of a patch of homogeneous Cauchy material against the applied stress gradient (equivalent couple). But the RVE size R is related to inclusion size a by volume fraction f. When f approaches zero, in fact a finite RVE size cannot be hold, hence we get a micropolar continuum with infinitesimal higher-order constants, which is indeed a classical medium.

3.3. Cubic displacement boundary condition

The following cubic displacement boundary conditions for the cylindrical RVE are applied:

$$u_1|_{\partial V} = \frac{2}{R^2}(y^3 - 3x^2y)(E_{\langle 12\rangle} - E_{\langle 21\rangle}), \tag{32a}$$

$$u_2|_{\partial V} = \frac{2}{R^2}(-x^3 + 3xy^2)(E_{\langle 12\rangle} - E_{\langle 21\rangle}), \tag{32b}$$

where $E_{\langle 12\rangle}$ and $E_{\langle 21\rangle}$ are the antisymmetric components of strain. The stress function F_3^i corresponding to the cubic displacements is adopted here

$$F_3^i = (A_1^i r^4 + A_2^i r^6 + A_3^i r^{-4} + A_4^i r^{-2})\cos(4\theta) + (A_5^i r^4 + A_6^i r^6 + A_7^i r^{-4} + A_8^i r^{-2})\sin(4\theta). \tag{33}$$

By using the same idea, only considering cubic boundary conditions, only non-zero component D_{1212} is derived for the heterogeneous material, which is called as anti-symmetric shear moduli κ in micropolar theory

$$\kappa = E(\varphi_1(v) + \varphi_2(v, \eta_1, \eta_2)f^3 + O(f^4)), \tag{34}$$

where $\varphi_1(v)$ and $\varphi_2(v, \eta_1, \eta_2)$ are non-dimensional functions.

For porous materials,

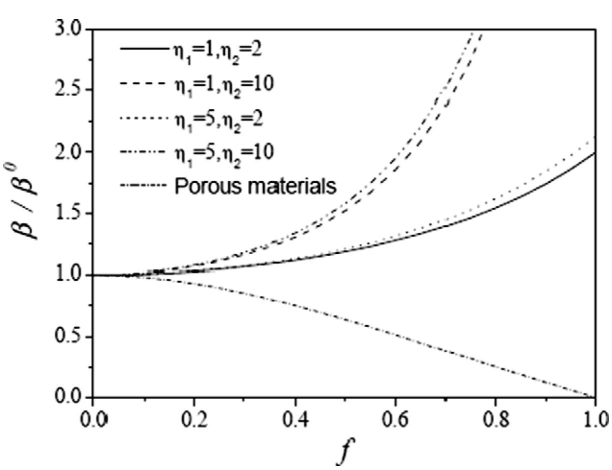

Fig. 1. Variations of β/β^0 as a function of f. Generally the normalized modulus increases with f for $\eta_2 > 1$ and is dominated by η_2. For softened inclusions ($\eta_2 < 1$) the modulus decreases with f.

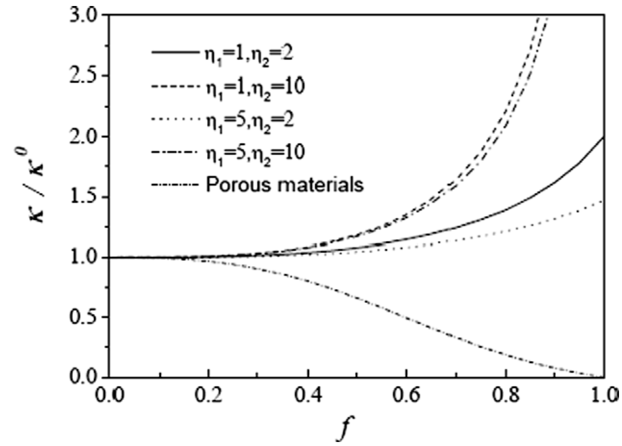

Fig. 2. Variations of κ/κ^0 as a function of f. The normalized modulus increases with f and η_2, but decreases with η_1.

$$\kappa^p = \frac{6E}{1+v} - \frac{8(9+5v)}{3(1+v)^3}f^3 + O(f^4). \tag{35}$$

From Fig. 2, it can be seen that for a general composite, the normalized κ/κ^0 increases with f and η_2, but decreases with η_1, where $\kappa^0 = 6E/(1+v)$ is the corresponding quantity for the matrix material. For porous materials, the normalized κ/κ^0 decreases with the increase of f. When f tends to zero, both κ and κ^p return to κ^0.

As can be seen the micropolar effect is pronounced whenever the inclusion properties differ from the matrix. This is different with results obtained by Bigoni and Drugan [7], where higher-order effect enters only for inclusions less stiff than the matrix. This might be attributed to the different formulation and macroscopic homogenized media employed. In their paper [7], a quadratic background displacement was considered, and then a presumed homogeneous couple-stress (constrained micropolar) media can be solved under the same boundary displacement as a Cauchy material. This is not the case for the present paper, which admits a cubic displacement and full micropolar is obtained.

4. Conclusions

In the present paper an analytical method to identify material parameters of micropolar theory for a composite material is proposed. Based on quadratic and cubic displacement boundary conditions on a cylindrical RVE, the analytical relations between the higher-order moduli and the microstructure of the composite are derived.

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